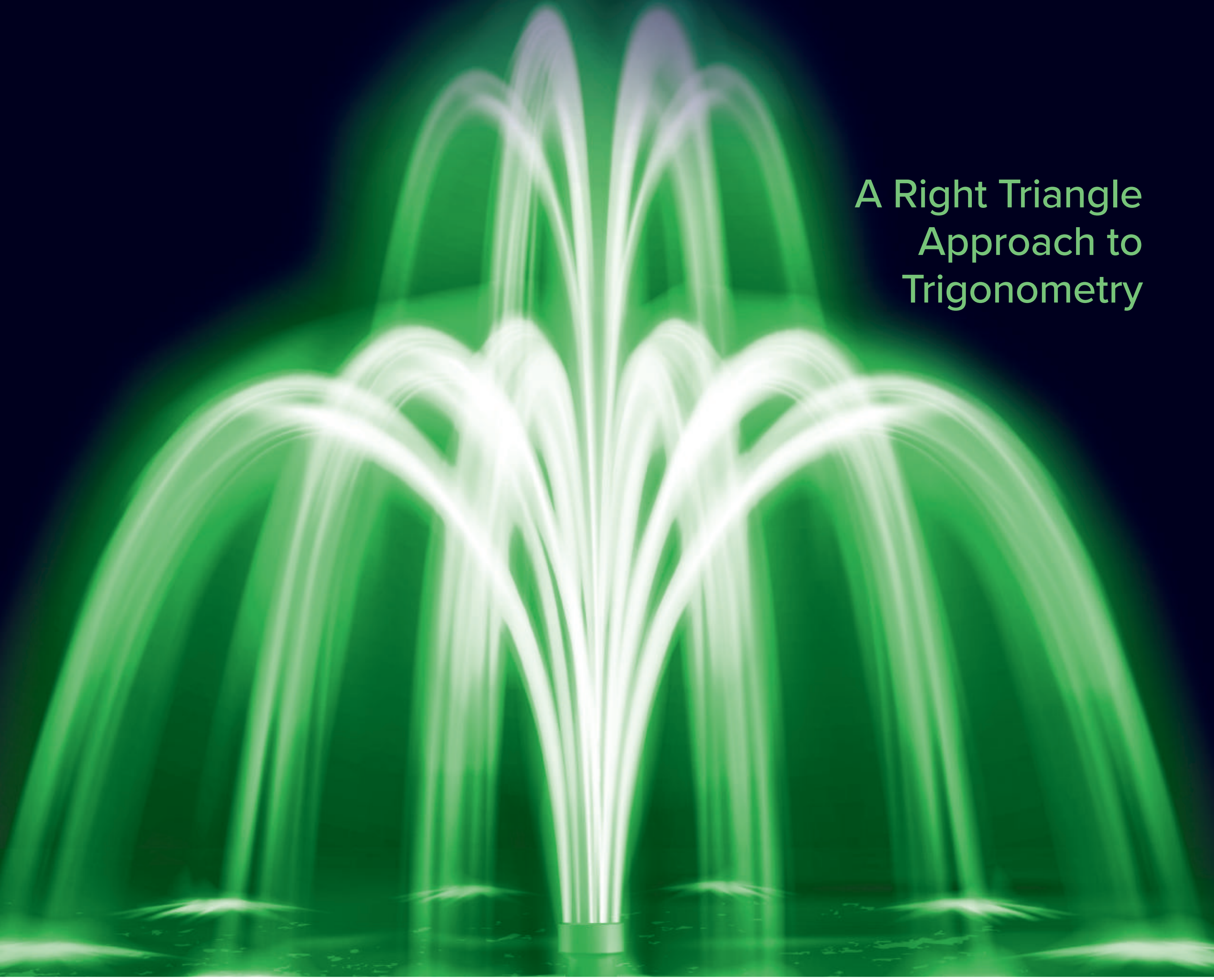
PRECALCULUS
CONCEPTS THROUGH FUNCTIONS

5th edition



A Right Triangle Approach to Trigonometry

Sullivan & Sullivan

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Precalculus

CONCEPTS THROUGH FUNCTIONS

A Right Triangle Approach To Trigonometry

Fifth Edition

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Pearson

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For the Next Generation

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Contents

To the Student xvi

The Concepts Through Functions Series xvii

Preface to the Instructor xviii

Applications Index xxvii

F Foundations: A Prelude to Functions 1

F.1 The Distance and Midpoint Formulas 2
Use the Distance Formula • Use the Midpoint Formula

F.2 Graphs of Equations in Two Variables; Intercepts; Symmetry 9
Graph Equations by Plotting Points • Find Intercepts from a Graph • Find Intercepts from an Equation • Test an Equation for Symmetry with Respect to the x-Axis, the y-Axis, and the Origin • Know How to Graph Key Equations

F.3 Lines 21
Calculate and Interpret the Slope of a Line • Graph Lines Given a Point and the Slope • Find the Equation of a Vertical Line • Use the Point-Slope Form of a Line; Identify Horizontal Lines • Use the Slope-Intercept Form of a Line • Find the Equation of a Line Given Two Points • Graph Lines Written in General Form Using Intercepts • Find Equations of Parallel Lines • Find Equations of Perpendicular Lines

F.4 Circles 36
Write the Standard Form of the Equation of a Circle • Graph a Circle • Work with the General Form of the Equation of a Circle

Chapter Project 44

1 Functions and Their Graphs 45

1.1 Functions 46
Describe a Relation • Determine Whether a Relation Represents a Function • Use Function Notation; Find the Value of a Function • Find the Difference Quotient of a Function • Find the Domain of a Function Defined by an Equation • Form the Sum, Difference, Product, and Quotient of Two Functions

1.2 The Graph of a Function 62
Identify the Graph of a Function • Obtain Information from or about the Graph of a Function

1.3 Properties of Functions 71
Identify Even and Odd Functions from a Graph • Identify Even and Odd Functions from an Equation • Use a Graph to Determine Where a Function Is Increasing, Decreasing, or Constant • Use a Graph to Locate Local Maxima and Local Minima • Use a Graph to Locate the Absolute Maximum and the Absolute Minimum • Use a Graphing Utility to Approximate Local Maxima and Local Minima and to Determine Where a Function Is Increasing or Decreasing • Find the Average Rate of Change of a Function

1.4 Library of Functions; Piecewise-defined Functions 85
Graph the Functions Listed in the Library of Functions • Analyze a Piecewise-defined Function

1.5 Graphing Techniques: Transformations 96
Graph Functions Using Vertical and Horizontal Shifts • Graph Functions Using Compressions and Stretches • Graph Functions Using Reflections About the x-Axis and the y-Axis

1.6 Building Mathematical Models from Verbal Descriptions 111
Build and Analyze Functions

1.7 Building Mathematical Models Using Variation 116
Construct a Model Using Direct Variation • Construct a Model Using Inverse Variation • Construct a Model Using Joint Variation or Combined Variation

Chapter Review 122

Chapter Test 126

Chapter Projects 127

2 Linear and Quadratic Functions 129

2.1 Properties of Linear Functions and Linear Models 130
Graph Linear Functions • Use Average Rate of Change to Identify Linear Functions • Determine Whether a Linear Function Is Increasing, Decreasing, or Constant • Find the Zero of a Linear Function • Build Linear Models from Verbal Descriptions

2.2 Building Linear Models from Data 141
Draw and Interpret Scatter Plots • Distinguish Between Linear and Nonlinear Relations • Use a Graphing Utility to Find the Line of Best Fit

2.3 Quadratic Functions and Their Zeros 149
Find the Zeros of a Quadratic Function by Factoring • Find the Zeros of a Quadratic Function Using the Square Root Method • Find the Zeros of a Quadratic Function by Completing the Square • Find the Zeros of a Quadratic Function Using the Quadratic Formula • Find the Point of Intersection of Two Functions • Solve Equations That Are Quadratic in Form

2.4 Properties of Quadratic Functions 161
Graph a Quadratic Function Using Transformations • Identify the Vertex and Axis of Symmetry of a Parabola • Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts • Find a Quadratic Function Given Its Vertex and One Other Point • Find the Maximum or Minimum Value of a Quadratic Function

2.5 Inequalities Involving Quadratic Functions 175
Solve Inequalities Involving a Quadratic Function

2.6 Building Quadratic Models from Verbal Descriptions and from Data 179
Build Quadratic Models from Verbal Descriptions • Build Quadratic Models from Data

2.7 Complex Zeros of a Quadratic Function 187
Find the Complex Zeros of a Quadratic Function

2.8 Equations and Inequalities Involving the Absolute Value Function 191
Solve Absolute Value Equations • Solve Absolute Value Inequalities

Chapter Review 196

Chapter Test 200

Cumulative Review 201

Chapter Projects 202

3 Polynomial and Rational Functions 203

3.1 Polynomial Functions 204
Identify Polynomial Functions and Their Degree • Graph Polynomial Functions Using Transformations • Identify the Real Zeros of a Polynomial Function and Their Multiplicity

3.2 Graphing Polynomial Functions; Models 220
Graph a Polynomial Function • Graph a Polynomial Function Using a Graphing Utility • Build Cubic Models from Data

3.3 The Real Zeros of a Polynomial Function 227
Use the Remainder and Factor Theorems • Use Descartes' Rule of Signs to Determine the Number of Positive and the Number of Negative Real Zeros of a Polynomial Function • Use the Rational Zeros Theorem to List the Potential Rational Zeros of a Polynomial Function • Find the Real Zeros of a Polynomial Function • Solve Polynomial Equations • Use the Theorem for Bounds on Zeros • Use the Intermediate Value Theorem

3.4 Complex Zeros; Fundamental Theorem of Algebra 241
Use the Conjugate Pairs Theorem • Find a Polynomial Function with Specified Zeros • Find the Complex Zeros of a Polynomial Function

3.5 Properties of Rational Functions 248
Find the Domain of a Rational Function • Find the Vertical Asymptotes of a Rational Function • Find a Horizontal or an Oblique Asymptote of a Rational Function

3.6 The Graph of a Rational Function 260
Graph a Rational Function • Solve Applied Problems Involving Rational Functions

3.7 Polynomial and Rational Inequalities 275
Solve Polynomial Inequalities • Solve Rational Inequalities

Chapter Review 282

Chapter Test 285

Cumulative Review 285

Chapter Projects 286

4 Exponential and Logarithmic Functions 288

4.1 Composite Functions 289
Form a Composite Function • Find the Domain of a Composite Function

4.2 One-to-One Functions; Inverse Functions 297
Determine Whether a Function Is One-to-One • Determine the Inverse of a Function Defined by a Mapping or a Set of Ordered Pairs • Obtain the Graph of the Inverse Function from the Graph of a One-to-One Function • Verify that a Function Defined by an Equation Is an Inverse Function • Find the Inverse of a Function Defined by an Equation

4.3 Exponential Functions 310
Evaluate Exponential Functions • Graph Exponential Functions • Define the Number e • Solve Exponential Equations

4.4 Logarithmic Functions 328
Change Exponential Statements to Logarithmic Statements and Logarithmic Statements to Exponential Statements • Evaluate Logarithmic Expressions • Determine the Domain of a Logarithmic Function • Graph Logarithmic Functions • Solve Logarithmic Equations

4.5 Properties of Logarithms 342
Work with the Properties of Logarithms • Write a Logarithmic Expression as a Sum or Difference of Logarithms • Write a Logarithmic Expression as a Single Logarithm • Evaluate Logarithms Whose Base Is Neither 10 Nor *e* • Graph a Logarithmic Function Whose Base Is Neither 10 Nor *e*

4.6 Logarithmic and Exponential Equations 351
Solve Logarithmic Equations • Solve Exponential Equations • Solve Logarithmic and Exponential Equations Using a Graphing Utility

4.7 Financial Models 358
Determine the Future Value of a Lump Sum of Money • Calculate Effective Rates of Return • Determine the Present Value of a Lump Sum of Money • Determine the Rate of Interest or the Time Required to Double a Lump Sum of Money

4.8 Exponential Growth and Decay Models; Newton's Law; Logistic Growth and Decay Models 368
Model Populations That Obey the Law of Uninhibited Growth • Model Populations That Obey the Law of Uninhibited Decay • Use Newton's Law of Cooling • Use Logistic Models

4.9 Building Exponential, Logarithmic, and Logistic Models from Data 379
Build an Exponential Model from Data • Build a Logarithmic Model from Data • Build a Logistic Model from Data

Chapter Review 386

Chapter Test 391

Cumulative Review 392

Chapter Projects 393

5 Trigonometric Functions 394

5.1 Angles, Arc Length, and Circular Motion 395
Angles and Degree Measure • Convert Between Decimal and Degree, Minute, Second Measures for Angles • Find the Length of an Arc of a Circle • Convert from Degrees to Radians and from Radians to Degrees • Find the Area of a Sector of a Circle • Find the Linear Speed of an Object Traveling in Circular Motion

5.2 Right Triangle Trigonometry 408
Find the Values of Trigonometric Functions of Acute Angles • Use Fundamental Identities • Find the Values of the Remaining Trigonometric Functions, Given the Value of One of Them • Use the Complementary Angle Theorem

5.3 Computing the Values of Trigonometric Functions of Acute Angles 421
Find the Exact Values of the Trigonometric Functions of $\frac{\pi}{4} = 45°$ • Find the Exact Values of the Trigonometric Functions of $\frac{\pi}{6} = 30°$ and $\frac{\pi}{3} = 60°$ • Use a Calculator to Approximate the Values of the Trigonometric Functions of Acute Angles • Model and Solve Applied Problems Involving Right Triangles

5.4 Trigonometric Functions of Any Angle 432
Find the Exact Values of the Trigonometric Functions for Any Angle • Use Coterminal Angles to Find the Exact Value of a Trigonometric Function • Determine the Signs of the Trigonometric Functions of an Angle in a Given Quadrant • Find the Reference Angle of an Angle • Use a Reference Angle to Find the Exact Value of a Trigonometric Function • Find the Exact Values of the Trigonometric Functions of an Angle, Given Information about the Functions

5.5 Unit Circle Approach; Properties of the Trigonometric Functions 443
Find the Exact Values of the Trigonometric Functions Using the Unit Circle • Know the Domain and Range of the Trigonometric Functions • Use Periodic Properties to Find the Exact Values of the Trigonometric Functions • Use Even–Odd Properties to Find the Exact Values of the Trigonometric Functions

5.6 Graphs of the Sine and Cosine Functions 456
Graph the Sine Function $y = \sin x$ and Functions of the Form $y = A\sin(\omega x)$ • Graph the Cosine Function $y = \cos x$ and Functions of the Form $y = A\cos(\omega x)$ • Determine the Amplitude and Period of Sinusoidal Functions • Graph Sinusoidal Functions Using Key Points • Find an Equation for a Sinusoidal Graph

5.7 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions 472
Graph the Tangent Function $y = \tan x$ and the Cotangent Function $y = \cot x$ • Graph Functions of the Form $y = A\tan(\omega x) + B$ and $y = A\cot(\omega x) + B$ • Graph the Cosecant Function $y = \csc x$ and the Secant Function $y = \sec x$ • Graph Functions of the Form $y = A\csc(\omega x) + B$ and $y = A\sec(\omega x) + B$

5.8 Phase Shift; Sinusoidal Curve Fitting 480
Graph Sinusoidal Functions of the Form $y = A\sin(\omega x - \phi) + B$ • Build Sinusoidal Models from Data

Chapter Review 491

Chapter Test 497

Cumulative Review 497

Chapter Projects 498

6 Analytic Trigonometry 500

6.1 The Inverse Sine, Cosine, and Tangent Functions 501
Define the Inverse Sine Function • Find the Value of an Inverse Sine Function • Define the Inverse Cosine Function • Find the Value of an Inverse Cosine Function • Define the Inverse Tangent Function • Find the Value of an Inverse Tangent Function • Use Properties of Inverse Functions to Find Exact Values of Certain Composite Functions • Find the Inverse Function of a Trigonometric Function • Solve Equations Involving Inverse Trigonometric Functions

6.2 The Inverse Trigonometric Functions (Continued) 515
Define the Inverse Secant, Cosecant, and Cotangent Functions • Find the Value of Inverse Secant, Cosecant, and Cotangent Functions • Find the Exact Value of Composite Functions Involving the Inverse Trigonometric Functions • Write a Trigonometric Expression as an Algebraic Expression

6.3 Trigonometric Equations 521
Solve Equations Involving a Single Trigonometric Function • Solve Trigonometric Equations Using a Calculator • Solve Trigonometric Equations Quadratic in Form • Solve Trigonometric Equations Using Fundamental Identities • Solve Trigonometric Equations Using a Graphing Utility

6.4 Trigonometric Identities 531
Use Algebra to Simplify Trigonometric Expressions • Establish Identities

6.5 Sum and Difference Formulas 539
Use Sum and Difference Formulas to Find Exact Values • Use Sum and Difference Formulas to Establish Identities • Use Sum and Difference Formulas Involving Inverse Trigonometric Functions • Solve Trigonometric Equations Linear in Sine and Cosine

6.6 Double-angle and Half-angle Formulas 552
Use Double-angle Formulas to Find Exact Values • Use Double-angle Formulas to Establish Identities • Use Half-angle Formulas to Find Exact Values

6.7 Product-to-Sum and Sum-to-Product Formulas 563
Express Products as Sums • Express Sums as Products

Chapter Review 567

Chapter Test 570

Cumulative Review 571

Chapter Projects 572

7 Applications of Trigonometric Functions 573

7.1 Applications Involving Right Triangles 574
Solve Right Triangles • Solve Applied Problems

7.2 The Law of Sines 580
Solve SAA or ASA Triangles • Solve SSA Triangles • Solve Applied Problems

7.3 The Law of Cosines 591
Solve SAS Triangles • Solve SSS Triangles • Solve Applied Problems

7.4 Area of a Triangle 598
Find the Area of SAS Triangles • Find the Area of SSS Triangles

7.5 Simple Harmonic Motion; Damped Motion; Combining Waves 604
Build a Model for an Object in Simple Harmonic Motion • Analyze Simple Harmonic Motion • Analyze an Object in Damped Motion • Graph the Sum of Two Functions

Chapter Review 614

Chapter Test 616

Cumulative Review 617

Chapter Projects 618

8 Polar Coordinates; Vectors 620

8.1 Polar Coordinates 621
Plot Points Using Polar Coordinates • Convert from Polar Coordinates to Rectangular Coordinates • Convert from Rectangular Coordinates to Polar Coordinates • Transform Equations Between Polar and Rectangular Forms

8.2 Polar Equations and Graphs 630
Identify and Graph Polar Equations by Converting to Rectangular Equations • Test Polar Equations for Symmetry • Graph Polar Equations by Plotting Points

8.3 The Complex Plane; De Moivre's Theorem 646
Plot Points in the Complex Plane • Convert a Complex Number Between Rectangular Form and Polar Form or Exponential Form • Find Products and Quotients of Complex Numbers • Use De Moivre's Theorem • Find Complex Roots

8.4 Vectors 655
Graph Vectors • Find a Position Vector • Add and Subtract Vectors Algebraically • Find a Scalar Multiple and the Magnitude of a Vector • Find a Unit Vector • Find a Vector from Its Direction and Magnitude • Model with Vectors

8.5 The Dot Product 670
Find the Dot Product of Two Vectors • Find the Angle Between Two Vectors • Determine Whether Two Vectors Are Parallel • Determine Whether Two Vectors Are Orthogonal • Decompose a Vector into Two Orthogonal Vectors • Compute Work

8.6 Vectors in Space 677
Find the Distance Between Two Points in Space • Find Position Vectors in Space • Perform Operations on Vectors • Find the Dot Product • Find the Angle Between Two Vectors • Find the Direction Angles of a Vector

8.7 The Cross Product 687
Find the Cross Product of Two Vectors • Know Algebraic Properties of the Cross Product • Know Geometric Properties of the Cross Product • Find a Vector Orthogonal to Two Given Vectors • Find the Area of a Parallelogram

Chapter Review 693

Chapter Test 696

Cumulative Review 697

Chapter Projects 697

9 Analytic Geometry 698

9.1 Conics 699
Know the Names of the Conics

9.2 The Parabola 700
Analyze Parabolas with Vertex at the Origin • Analyze Parabolas with Vertex at (h, k) • Solve Applied Problems Involving Parabolas

9.3 The Ellipse 709
Analyze Ellipses with Center at the Origin • Analyze Ellipses with Center at (h, k) • Solve Applied Problems Involving Ellipses

9.4 The Hyperbola 720
Analyze Hyperbolas with Center at the Origin • Find the Asymptotes of a Hyperbola • Analyze Hyperbolas with Center at (h, k) • Solve Applied Problems Involving Hyperbolas

9.5 Rotation of Axes; General Form of a Conic 733
Identify a Conic • Use a Rotation of Axes to Transform Equations • Analyze an Equation Using a Rotation of Axes • Identify Conics Without Rotating the Axes

9.6 Polar Equations of Conics 741
Analyze and Graph Polar Equations of Conics • Convert the Polar Equation of a Conic to a Rectangular Equation

9.7 Plane Curves and Parametric Equations 748
Graph Parametric Equations • Find a Rectangular Equation for a Plane Curve Defined Parametrically • Use Time as a Parameter in Parametric Equations • Find Parametric Equations for Plane Curves Defined by Rectangular Equations

Chapter Review 761

Chapter Test 763

Cumulative Review 764

Chapter Projects 764

10 Systems of Equations and Inequalities 766

10.1 Systems of Linear Equations: Substitution and Elimination 767
Solve Systems of Equations by Substitution • Solve Systems of Equations by Elimination • Identify Inconsistent Systems of Equations Containing Two Variables • Express the Solution of a System of Dependent Equations Containing Two Variables • Solve Systems of Three Equations Containing Three Variables • Identify Inconsistent Systems of Equations Containing Three Variables • Express the Solution of a System of Dependent Equations Containing Three Variables

10.2 Systems of Linear Equations: Matrices 781
Write the Augmented Matrix of a System of Linear Equations • Write the System of Equations from the Augmented Matrix • Perform Row Operations on a Matrix • Solve a System of Linear Equations Using Matrices

10.3 Systems of Linear Equations: Determinants 796
Evaluate 2 by 2 Determinants • Use Cramer's Rule to Solve a System of Two Equations Containing Two Variables • Evaluate 3 by 3 Determinants • Use Cramer's Rule to Solve a System of Three Equations Containing Three Variables • Know Properties of Determinants

10.4 Matrix Algebra 807
Find the Sum and Difference of Two Matrices • Find Scalar Multiples of a Matrix • Find the Product of Two Matrices • Find the Inverse of a Matrix • Solve a System of Linear Equations Using an Inverse Matrix

10.5 Partial Fraction Decomposition 824
Decompose $\frac{P}{Q}$ Where Q Has Only Nonrepeated Linear Factors • Decompose $\frac{P}{Q}$ Where Q Has Repeated Linear Factors • Decompose $\frac{P}{Q}$ Where Q Has a Nonrepeated Irreducible Quadratic Factor • Decompose $\frac{P}{Q}$ Where Q Has a Repeated Irreducible Quadratic Factor

10.6 Systems of Nonlinear Equations 833
Solve a System of Nonlinear Equations Using Substitution • Solve a System of Nonlinear Equations Using Elimination

10.7 Systems of Inequalities 842
Graph an Inequality • Graph a System of Inequalities

10.8 Linear Programming 850
Set Up a Linear Programming Problem • Solve a Linear Programming Problem

Chapter Review 857

Chapter Test 860

Cumulative Review 861

Chapter Projects 862

11 Sequences; Induction; the Binomial Theorem 863

11.1 Sequences 864
List the First Several Terms of a Sequence • List the Terms of a Sequence Defined by a Recursive Formula • Use Summation Notation • Find the Sum of a Sequence

11.2 Arithmetic Sequences 874
Determine Whether a Sequence Is Arithmetic • Find a Formula for an Arithmetic Sequence • Find the Sum of an Arithmetic Sequence

11.3 Geometric Sequences; Geometric Series 881
Determine Whether a Sequence Is Geometric • Find a Formula for a Geometric Sequence • Find the Sum of a Geometric Sequence • Determine Whether a Geometric Series Converges or Diverges • Solve Annuity Problems

11.4 Mathematical Induction 893
Prove Statements Using Mathematical Induction

11.5 The Binomial Theorem 897
Evaluate $\binom{n}{j}$ • Use the Binomial Theorem

Chapter Review 903
Chapter Test 906
Cumulative Review 906
Chapter Projects 907

12 Counting and Probability 908

12.1 Counting 909
Find All the Subsets of a Set • Count the Number of Elements in a Set • Solve Counting Problems Using the Multiplication Principle

12.2 Permutations and Combinations 914
Solve Counting Problems Using Permutations Involving n Distinct Objects • Solve Counting Problems Using Combinations • Solve Counting Problems Using Permutations Involving n Nondistinct Objects

12.3 Probability 923
Construct Probability Models • Compute Probabilities of Equally Likely Outcomes • Find Probabilities of the Union of Two Events • Use the Complement Rule to Find Probabilities

Chapter Review 933
Chapter Test 935
Cumulative Review 936
Chapter Projects 936

13 A Preview of Calculus: The Limit, Derivative, and Integral of a Function 938

13.1 Investigating Limits Using Tables and Graphs 939
Investigate a Limit Using a Table • Investigate a Limit Using a Graph

13.2 Algebraic Techniques for Finding Limits 944
Find the Limit of a Sum, a Difference, and a Product • Find the Limit of a Polynomial • Find the Limit of a Power or a Root • Find the Limit of a Quotient • Find the Limit of an Average Rate of Change

13.3 One-sided Limits; Continuity 951
Find the One-sided Limits of a Function • Determine Whether a Function Is Continuous at a Number

13.4 The Tangent Problem; The Derivative 957
Find an Equation of the Tangent Line to the Graph of a Function • Find the Derivative of a Function • Find Instantaneous Rates of Change • Find the Instantaneous Velocity of an Object

13.5 The Area Problem; The Integral 965
Approximate the Area Under the Graph of a Function • Approximate Integrals Using a Graphing Utility

Chapter Review 971

Chapter Test 974

Chapter Projects 975

Appendix A Review A1

A.1 Algebra Essentials A1
Work with Sets • Graph Inequalities • Find Distance on the Real Number Line • Evaluate Algebraic Expressions • Determine the Domain of a Variable • Use the Laws of Exponents • Evaluate Square Roots • Use a Calculator to Evaluate Exponents

A.2 Geometry Essentials A14
Use the Pythagorean Theorem and Its Converse • Know Geometry Formulas • Understand Congruent Triangles and Similar Triangles

A.3 Polynomials A23
Recognize Monomials • Recognize Polynomials • Add and Subtract Polynomials • Multiply Polynomials • Know Formulas for Special Products • Divide Polynomials Using Long Division • Work with Polynomials in Two Variables

A.4 Factoring Polynomials A33
Factor the Difference of Two Squares and the Sum and Difference of Two Cubes • Factor Perfect Squares • Factor a Second-degree Polynomial: $x^2 + Bx + C$ • Factor by Grouping • Factor a Second-degree Polynomial: $Ax^2 + Bx + C, A \neq 1$ • Complete the Square

A.5 Synthetic Division A43
Divide Polynomials Using Synthetic Division

A.6 Rational Expressions A47
Reduce a Rational Expression to Lowest Terms • Multiply and Divide Rational Expressions • Add and Subtract Rational Expressions • Use the Least Common Multiple Method • Simplify Complex Rational Expressions

A.7 *n*th Roots; Rational Exponents A57
Work with *n*th Roots • Simplify Radicals • Rationalize Denominators and Numerators • Simplify Expressions with Rational Exponents

A.8 Solving Equations A66
Solve Linear Equations • Solve Rational Equations • Solve Equations by Factoring • Solve Radical Equations

A.9 Problem Solving: Interest, Mixture, Uniform Motion, Constant Rate Job Applications A75
Translate Verbal Descriptions into Mathematical Expressions • Solve Interest Problems • Solve Mixture Problems • Solve Uniform Motion Problems • Solve Constant Rate Job Problems

A.10 Interval Notation; Solving Inequalities A84
Use Interval Notation • Use Properties of Inequalities • Solve Inequalities • Solve Combined Inequalities

A.11 Complex Numbers A92
Add, Subtract, Multiply, and Divide Complex Numbers

Appendix B Graphing Utilities B1

B.1 The Viewing Rectangle B1

B.2 Using a Graphing Utility to Graph Equations B3

B.3 Using a Graphing Utility to Locate Intercepts and Check for Symmetry B5

B.4 Using a Graphing Utility to Solve Equations B6

B.5 Square Screens B8

B.6 Using a Graphing Utility to Graph Inequalities B9

B.7 Using a Graphing Utility to Solve Systems of Linear Equations B9

B.8 Using a Graphing Utility to Graph a Polar Equation B11

B.9 Using a Graphing Utility to Graph Parametric Equations B11

Answers AN1

Challenge Problem Solutions CP1

Subject Index I1

To the Student

As you begin, you may feel anxious about the number of theorems, definitions, procedures, and equations. You may wonder if you can learn it all in time. Don't worry, your concerns are normal. This textbook was written with you in mind. If you attend class, work hard, and read and study this book, you will build the knowledge and skills you need to be successful. Here's how you can use the book to your benefit.

Read Carefully

When you get busy, it's easy to skip reading and go right to the problems. Don't . . . the book has a large number of examples and clear explanations to help you break down the mathematics into easy-to-understand steps. Reading will provide you with a clearer understanding, beyond simple memorization. Read before class (not after) so you can ask questions about anything you didn't understand. You'll be amazed at how much more you'll get out of class if you do this.

Use the Features

We use many different methods in the classroom to communicate. Those methods, when incorporated into the book, are called "features." The features serve many purposes, from providing timely review of material you learned before (just when you need it), to providing organized review sessions to help you prepare for quizzes and tests. Take advantage of the features and you will master the material.

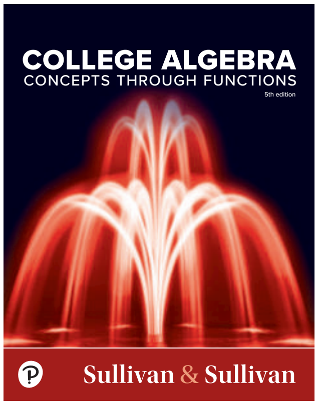
To make this easier, we've provided a brief guide to getting the most from this book. Refer to the "Prepare for Class," "Practice," and "Review" at the front of the book. Spend fifteen minutes reviewing the guide and familiarizing yourself with the features by flipping to the page numbers provided. Then, as you read, use them. This is the best way to make the most of your textbook.

Please do not hesitate to contact us, through Pearson Education, with any questions, suggestions, or comments that would improve this text. We look forward to hearing from you, and good luck with all of your studies.

Best Wishes!

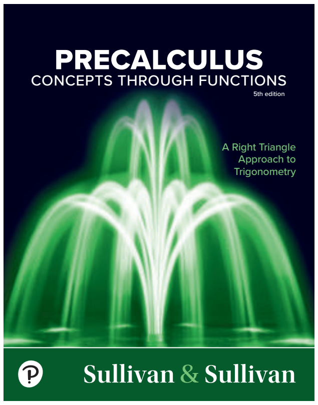
Michael Sullivan
Michael Sullivan, III

The Concepts Through Functions Series



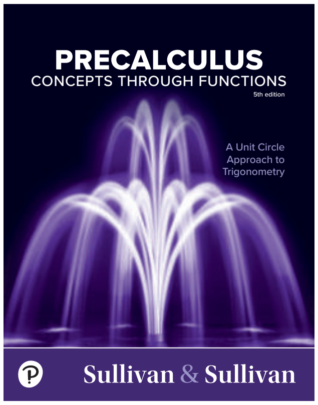
College Algebra, Fifth Edition

This text provides an approach to college algebra that introduces functions early (Chapter 1). All material is presented through the eyes of a function. So, rather than have a chapter dedicated to review, the material is presented from a function point of view. For example, rather than reviewing the various approaches to solving quadratic equations, students are asked to find the zeroes of a quadratic function or the x-intercepts of the graph of a quadratic function. This allows for review of the concepts, but also requires students to solve equations in the form $f(x) = 0$, which foreshadows solving $f'(x) = 0$ in calculus. Graphing calculator, Desmos, and GeoGebra usage is provided, but optional. Examples that require graphing technology are marked with . Exercises that require graphing technology are indicated with . After completing this text, a student will be prepared for trigonometry, finite mathematics, and business calculus.



Precalculus, A Right Triangle Approach to Trigonometry, Fifth Edition

This text contains all the material in *College Algebra*, but it also develops the trigonometric functions using a right triangle approach and shows how that approach is related to the unit circle approach. Graphing techniques are emphasized, including a thorough discussion of polar coordinates, parametric equations, and conics using polar coordinates. Graphing calculator, Desmos, and GeoGebra usage is provided, but optional. The final chapter provides an introduction to calculus, with a discussion of the limit, the derivative, and the integral of a function. After completing this text, a student will be prepared for engineering calculus and business calculus.



Precalculus, A Unit Circle Approach to Trigonometry, Fifth Edition

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Preface to the Instructor

Having taught at an urban university (Michael Sullivan) and a community college (Michael Sullivan III), we are aware of the varied needs of students in this course. Such students range from those who have little mathematical background and are fearful of mathematics courses, to those with a strong mathematical education and a high level of motivation. For some of your students, this will be their last course in mathematics, whereas others will further their mathematical education. We have written this text with both groups in mind.

As a teacher, and as an author of precalculus, engineering calculus, finite mathematics, and business calculus texts, Michael Sullivan understands what students must know if they are to be focused and successful in upper-level math courses. As an instructor and an author of a developmental mathematics series, Michael's son and co-author, Michael Sullivan III, understands the trepidations and skills that students bring to precalculus. As the father of current college students, Michael III realizes that today's college students demand a variety of media to support their education. This text addresses that demand by providing technology, video, and personalized support in MyLab Math that enhances understanding without sacrificing math skills. Together, we have taken great pains to ensure that the text offers solid, student-friendly examples and problems, as well as a clear and seamless writing style.

A tremendous benefit of authoring a successful series is the broad-based feedback we receive from teachers and students. We are sincerely grateful for their support. Virtually every change in this edition is the result of their thoughtful comments and suggestions. We are confident that, building on the success of the first four editions and incorporating many of these suggestions, we have made *Precalculus: Concepts Through Functions, A Right Triangle Approach to Trigonometry,* 5th Edition, an even better tool for learning and teaching. We continue to encourage you to share with us your experiences teaching from this text.

Features in the Fifth Edition

A descriptive list of the many special features of this text can be found in the pages at the front of the book. This list places the features in their proper context, as building blocks of an overall learning system that has been carefully crafted over the years to help students get the most out of the time they put into studying. Please take the time to review this and to discuss it with your students at the beginning of your course. Our experience is that when students utilize these features, they are more successful in the course.

New to the Fifth Edition

New Within the Textbook

All of the exercises and examples in the text have been reviewed and analyzed, and we have incorporated feedback from users of the text. All time-sensitive problems have been updated to the most recent information available. Here are the new features of this edition:

- **Challenge Problems**—These problems appear in the Applications and Extensions part of the section exercises and are designed to challenge students. Full solutions are in the back of the Annotated Instructor's Edition and in the Instructor's Solution Manual.
- **"Need to Review?" Feature**—We placed reminders in the margin for key review topics. The reminders point students to the location of the review material in the textbook.
- **Chapter Projects**—The projects have been enhanced to give students an up-to-the-minute experience. Many of these projects require the student to research information online in order to solve problems.
- **Interactive Figure Exercises**—We have added this new category of exercises that require students to manipulate an interactive figure to solve. The interactive figures may be found at `bit.ly/3raFUGB` or in the Video & Resource Library of MyLab Math. They were created by author Michael Sullivan III in GeoGebra. These exercises are labeled with the icon .
- **Expanded! Retain Your Knowledge Problems**—These problems, which were new to the previous edition, are based on learning research, including a study of precalculus students at University of Louisville entitled "Spaced retrieval practice increases college students' short- and long-term retention of mathematics knowledge" (Hopkins et al, 2016). The Retain Your Knowledge problems were so well received that we have expanded them in this edition. Moreover, while the focus remains to help students maintain their skills, many problems were added that preview skills required to succeed in subsequent sections or in calculus ( ). All answers to Retain Your Knowledge problems are given in the back of the text and these problems are available in the prebuilt assignments in the Assignment Manager in MyLab Math.
- **Key to Exercise Types**—To help you navigate the features of the exercise sets, we've included a key at the bottom of the first page of each section's exercises.

1. Now Work 1. Modeling 1. Writing
1. Graphing Tech 1. Calculus Preview
1. Applet-Based

- **Graphing Utility Screen Captures**—In several instances we have added Desmos and GeoGebra screen captures along with the TI-84 Plus CE screen captures. These updated screen captures provide alternative ways of visualizing concepts and making connections between equations, data, and graphs in full color.
- **Diversity, Equity, and Inclusion**—Pearson conducted an external review of the text's content to determine how it could be improved to address issues related to diversity, equity, and inclusion. The results of that review informed this revision.

Content Changes

Chapter F

- NEW Section F.2 Example 9 Testing an Equation for Symmetry

Chapter 1

- NEW Section 1.1 Objective 1 Describe a Relation
  - NEW Example 1 Describing a Relation demonstrates using the Rule of Four to express a relation numerically, as a mapping, and graphically given a verbal description.
- NEW Section 1.2 Example 5 Expending Energy

Chapter 2

- Section 2.4 now introduces the concept of concavity for a quadratic function.
- NEW Section 2.4 Example 3 Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts
- NEW Section 2.4 Example 8 Analyzing the Motion of a Projectile (formerly in Section 2.6)
- NEW Section 2.5 Example 3 Solving an Inequality
- NEW Section 2.6 Example 4 Fitting a Quadratic Function to Data

Chapter 3

- Previous Section 3.1 has been revised and split into two sections:
  - 3.1 Polynomial Functions
  - 3.2 Graphing Polynomial Functions; Models
- NEW Section 3.2 Example 2 Graphing a Polynomial Function (a 4th degree polynomial function)

Chapter 4

- NEW Section 4.2 Objective: Verify a Function Defined by an Equation is an Inverse Function

Chapter 5

- NEW Section 5.1 Example 6 Field Width of a Digital Lens Reflex Camera Lens
- Sections 5.6 and 5.7 were reorganized for increased clarity. Two new objectives were added to Section 5.7.

Chapter 6

- Sections 6.1 and 6.2 were reorganized for increased clarity. Four new objectives were added to Section 6.1. The objectives in Section 6.2 were reordered.

Chapter 8

- Section 8.3 DeMoivre's Theorem was rewritten to support the exponential form of a complex number.
  - Euler's Formula is introduced to express a complex number in exponential form. The exponential form is used to compute products and quotients.
  - DeMoivre's Theorem is expressed using the exponential form of a complex number. The exponential form is used to find complex roots.

Chapter 10

- NEW Section 10.5 Example 1 Identifying Proper and Improper Rational Expressions

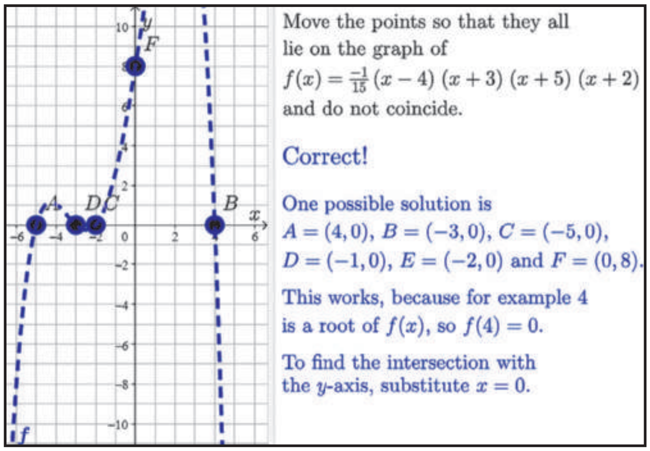
Appendix A

- Section A.7 Objective 3 now includes rationalizing the numerator. Problems 69–76 provide practice.

New Within MyLab Math

- **MyLab Exercises**—Author Michael Sullivan III reviewed all MyLab exercises for this revision and edited them to better match the language and approach of the text and to make the solutions (within *View an Example* and *Help Me Solve This*) more student-friendly.
- **Instructional Videos**—Every instructional video is new, created exclusively for this product! The videos have the following features:
  - **Author-driven**—The authors reviewed every script and every video to ensure the approach used in presenting material and solving problems reflects that in the text. In addition, the videos were reviewed by the authors for accuracy. Author Michael Sullivan III appears in many videos.
  - **Objective-based**—one video for each objective in the book
  - **Segmented**—videos divided into shorter parts for ease of navigation (Introductions, Examples, and Summary)
  - **Interactive Figures**—to help students visualize key concepts
  - **Handwriting**—our research showed that students prefer seeing examples worked out by hand
  - **High-definition**—clearly readable on phones
  - **Accessible**—all are close-captioned
  - **Various instructors**—Featured in the videos are:
    - Author Michael Sullivan III
    - Sue Glascoe (Mesa Community College)
    - Paulette Haywood-Watson (Stillman College)
    - Brian Macon (Valencia College)
    - Caleb Schroeder (Antelope Valley College)
- **Video Note-Taking Guide**—Helps students be active learners while watching videos. Written specifically for the new Sullivan video program by Kevin Bodden and Randy Gallaher at Lewis and Clark Community College.

- **Video Assignments**—These section-level assignments consist of short video clips followed by concept check and practice exercises. They are especially helpful for online classes or "flipped" classes, where some or all learning takes place independently.
- **Corequisite Support Resources**—Provide all the content and assessment resources necessary for students and instructors. MyLab Math supports various corequisite support models, including Concurrent (aka just-in-time) and Consecutive (aka front-loaded) models. For more details, see page xxi or the Corequisite Implementation Guide at `bit.ly/3ujaY9e`.
- **Interactive Figures** (formerly titled Guided Visualizations)—The suite of Interactive Figures has been expanded to support teaching and learning. The figures (created in GeoGebra, many by author Michael Sullivan III) illustrate key concepts and allow manipulation. They have been designed to be used in lecture as well as by students independently.



- ▲**GeoGebra Graphing Exercises**—Gradable graphing exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.
- **Enhanced Assignments**—These section-level assignments have three unique properties (and are fully editable).

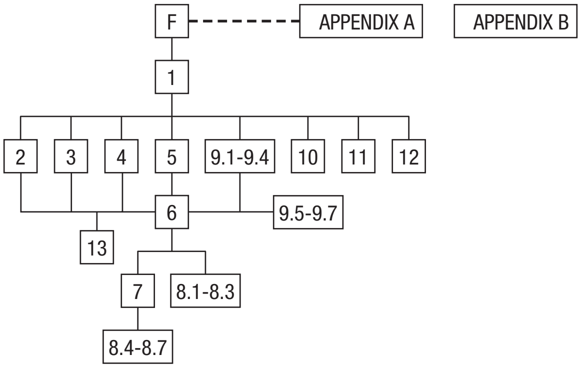
  (1) They help keep skills fresh with *spaced practice* of previously learned concepts. The spaced practice problems are selected from the Retain Your Knowledge problems in the textbook.

  (2) They have learning aids strategically turned off for some exercises to ensure that students understand how to work the exercises independently.

  (3) They contain personalized prerequisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.

Using the Fifth Edition Effectively with Your Syllabus

To meet the varied needs of diverse syllabi, this text contains more content than is likely to be covered in a typical precalculus course. As the chart illustrates, this text has been organized with flexibility of use in mind. Within a given chapter, certain sections are optional (see the details that follow the accompanying figure) and can be omitted without loss of continuity.



Foundations A Prelude to Functions
Quick coverage of this chapter, which is mainly review material, will enable you to get to Chapter 1, *Functions and Their Graphs*, earlier.

Chapter 1 Functions and Their Graphs
Perhaps the most important chapter. Sections 1.6 and 1.7 are optional.

Chapter 2 Linear and Quadratic Functions
Topic selection depends on your syllabus. Sections 2.2, 2.6, and 2.7 may be omitted without a loss of continuity.

Chapter 3 Polynomial and Rational Functions
Topic selection depends on your syllabus. Section 3.6 is optional.

Chapter 4 Exponential and Logarithmic Functions
Sections 4.1–4.6 follow in sequence. Sections 4.7–4.9 are optional.

Chapter 5 Trigonometric Functions
The sections follow in sequence. Section 5.8 is optional.

Chapter 6 Analytic Trigonometry
Sections 6.2 and 6.7 may be omitted in a brief course.

Chapter 7 Applications of Trigonometric Functions
Sections 7.4 and 7.5 may be omitted in a brief course.

Chapter 8 Polar Coordinates; Vectors
Sections 8.1–8.3 and Sections 8.4–8.7 are independent and may be covered separately.

Chapter 9 Analytic Geometry
Sections 9.1–9.4 follow in sequence. Sections 9.5, 9.6, and 9.7 are independent of each other, but each requires Sections 9.1–9.4.

Chapter 10 Systems of Equations and Inequalities
Sections 10.2–10.7 may be covered in any order. Section 10.8 requires Section 10.7.

Chapter 11 Sequences; Induction; the Binomial Theorem

There are three independent parts: Sections 11.1–11.3, Section 11.4, and Section 11.5.

Chapter 12 Counting and Probability

The sections follow in sequence.

Chapter 13 A Preview of Calculus: The Limit, Derivative, and Integral of a Function

If time permits, coverage of this chapter will provide your students with a beneficial head-start in calculus. The sections follow in sequence.

Appendix A Review

This review material may be covered at the start of a course or used as a just-in-time review. Specific references to this material occur throughout the text to assist in the review process.

Appendix B Graphing Utilities

Reference is made to these sections at the appropriate place in the text.

MyLab Math Resources for Success

MyLab Math (**pearson.com/mylab/math**) is available to accompany Pearson's market-leading text options, including this text (access code required). MyLab Math is the teaching and learning platform that empowers you to reach every student. It combines trusted author content—including full eText and online homework with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

NEW! Corequisite Course Support

This MyLab course supports various corequisite course models, including Concurrent (aka just-in-time) and Consecutive (aka front-loaded) models. The MyLab contains all of these learning and assessment resources to support corequisite courses:

1. **Complete Corequisite eText** built from Michael Sullivan III's developmental mathematics texts.
2. **Instructional videos** for each corequisite objective. Many of these videos feature Michael Sullivan III in the classroom.
3. **Assignable algorithmic exercises** for each corequisite objective.
4. **Worksheets** with instruction and exercises for each corequisite objective (also available in print).
5. **Activities** for selected corequisite objectives.
6. **Study Skills** videos for time management, mindset, stress management, college transition, and more.
7. **Corequisite Implementation Guide** with specific guidelines for using the materials to teach various corequisite models. See `bit.ly/3ujaY9e` to download this guide.

To help target instruction on corequisite objectives, MyLab includes these pre-made assessments:

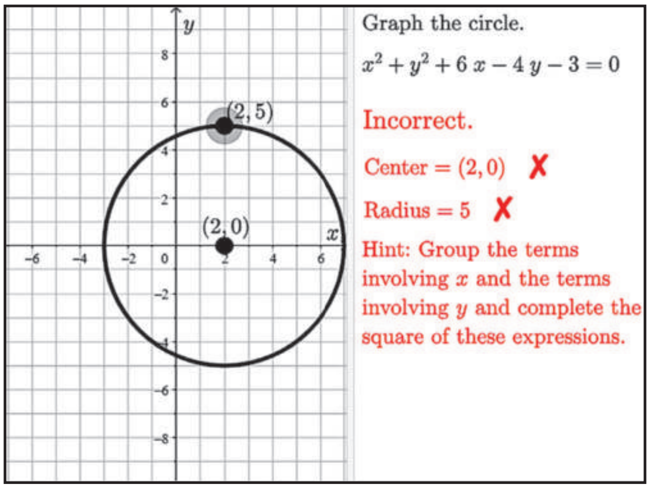
- **Pre-Course Quiz** addresses key arithmetic and basic algebra topics and is designed to be administered prior to beginning the course.
- **Skills Check Quiz** for each Chapter addresses the prerequisite skills needed for each chapter in precalculus.
- Based on the results of these quizzes, students can receive **personalized assignments** to address objectives that are not mastered. This way, students can focus on just the topics they need help with.

Note that the above resources are also designed to provide just-in-time help for students in your regular (non-corequisite) courses. (We understand that almost all students at some point need targeted refreshers on specific prerequisite skills.)

MyLab Math Student Resources

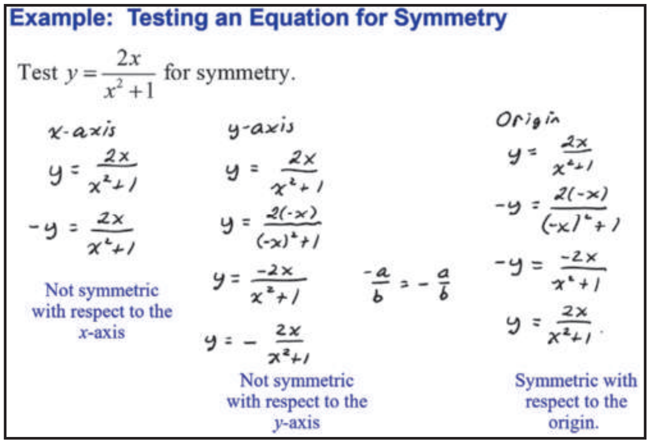
Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student needs practice, giving all students the support they need—when and where they need it—to be successful.

Exercises with Immediate Feedback—The exercises in MyLab Math reflect the approach and learning style of this text, and regenerate algorithmically to give students unlimited opportunity for practice and mastery. Most exercises include learning aids, such as guided solutions and sample problems, and they offer helpful feedback when students enter incorrect answers.



- NEW! ▲ **GeoGebra Exercises** are gradable graphing exercises that help students demonstrate their understanding. They enable students to interact directly with the graph in a manner that reflects how students would graph on paper.
- **Setup & Solve** exercises require students to first describe how they will set up and approach a problem. This reinforces conceptual understanding of the process

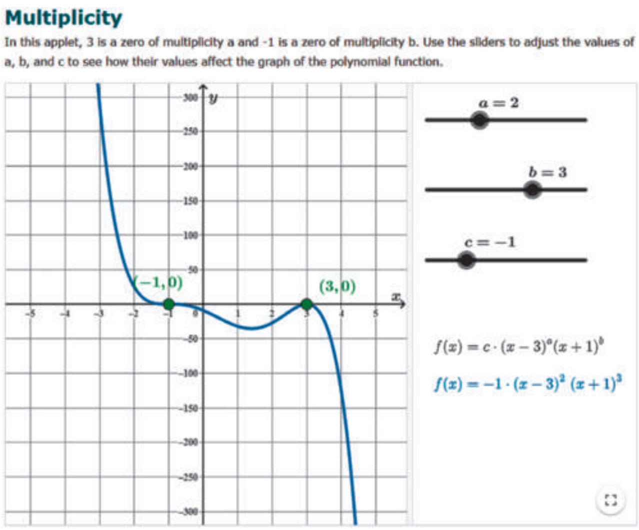
applied in approaching the problem, promotes long term retention of the skill, and mirrors what students will be expected to do on a test.



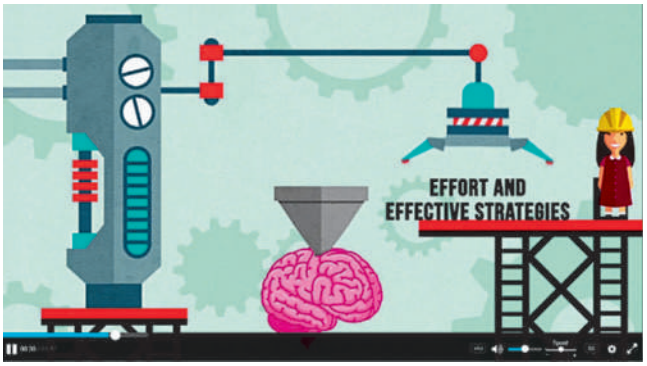
NEW! ▲**Instructional Videos**—Every instructional video is new, created exclusively for this product by experienced instructors (including author Michael Sullivan III). All videos were thoroughly vetted by the authors. See page xix for more details.

NEW! Video Note-Taking Guide—Helps students be active learners while watching videos. Written specifically for the new Sullivan video program by Kevin Bodden and Randy Gallaher at Lewis and Clark Community College.

Chapter Test Prep videos correspond to each exercise in the Chapter Test in the textbook, enabling students to effectively prepare for high-stakes testing. These are available in MyLab Math.



▲**Interactive Figures** (formerly titled Guided Visualizations) bring mathematical concepts to life, helping students see the concepts through directed explorations and purposeful manipulation. These figures, many created by author Michael Sullivan III, are assignable in MyLab Math and encourage active learning, critical thinking, and conceptual understanding. **NEW!** For this revision, we added many more interactive figures (in editable GeoGebra format) to the Video & Resource Library.



▲**Mindset videos** and assignable, open-ended exercises foster a growth mindset in students. This material encourages them to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as learning opportunities—so often a hurdle for math students.

Pearson eText—The eText is "reflowable" to adapt to use on tablets and smartphones. You can insert your own highlights, notes, and bookmarks. It is also fully accessible using screen-readers. Download the Pearson+ app to access your eText on your smartphone or tablet anytime—even offline.

NEW! Study Slides—PowerPoint slides designed for studying! Includes every section of the text. Fully screen-reader accessible.

Student Solutions Manual—Fully worked solutions to odd-numbered exercises. Available for download from within MyLab Math.

MyLab Math Instructor Resources

Your course is unique. So whether you'd like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

Pre-Built Assignments are designed to make the homework experience as effective as possible for students. All of these assignments are *fully editable*.

- **NEW! Enhanced Assignments**—These section-level assignments have three unique properties:

  (1) They help keep skills fresh with *spaced practice* of previously learned concepts. The spaced practice problems are selected from the Retain Your Knowledge problems in the textbook.

  (2) They have learning aids strategically turned off for some exercises to ensure that students understand how to work the exercises independently.

  (3) They contain personalized perquisite skills exercises for gaps identified in the chapter-level Skills Check Quiz.

- **NEW! Video Assignments**—These section-level assignments consist of short instructional videos followed by concept check and skill exercises. They are especially helpful for online classes or "flipped" classes, where some or all learning takes place independently.

Learning Catalytics—With Learning Catalytics™, you'll hear from every student when it matters most. You pose a variety of questions in class (choosing from pre-loaded questions—many written by author Michael Sullivan III—or questions of your own making) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops. For specifics on using Learning Catalytics for this text, see `bit.ly/3uiGOD1`.

Accessibility—Pearson works continuously to ensure our products are as accessible as possible to all students. Currently we work toward achieving WCAG 2.0 AA for our existing products (2.1 AA for future products) and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media (https://wps.pearsoned.com/accessibility/).

Other instructor resources include:

- **NEW! Annotated Instructor's Edition eText**—This page-for-page eText is available within the Instructor Resources section of MyLab Math.
- **Mini Lecture Notes** contain additional examples and helpful teaching tips for each section of the text.
- **Instructor Solution Manual** contains worked-out solutions for every exercise in the text.
- **PowerPoint Lecture Slides** are fully editable and included for each section of the text.
- **TestGen®** enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson's online catalog.
- **Test Bank** features printable PDFs containing all of the test exercises available in TestGen.

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Texts are written by authors, but they evolve from idea to final form through the efforts of many people.

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- Video creators: Sue Glascoe (Mesa Community College), Paulette Haywood-Watson (Stillman College), Brian Macon (Valencia College), Caleb Schroeder (Antelope Valley College)
- Video Note-Taking Guide writers Kevin Bodden and Randy Gallaher (Lewis & Clark Community College)
- Accuracy checkers: Jennifer Blue read the entire manuscript and checked the accuracy of answers. Timothy Britt created the Solutions Manuals and accuracy-checked answers.
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Mary Jule Gabiou, North Idaho College
Randy Gallaher, Lewis and Clark College
Tina Garn, University of Arizona
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Wayne Gibson, Rancho Santiago College
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Sudhir Kumar Goel, Valdosta State University
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Deltrye Holt, Augusta State University
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Pearson

Applications Index

Acoustics

amplifier output, 390
loudness of sound, 339
microphones, 20
whispering gallery, 715–716, 718

Aerodynamics

modeling aircraft motion, 620, 697

Agriculture

milk production, 384
number of farm workers, 377

Air travel

airline revenue, 856
bearing of an airplane, 576, 578
cost of parking at O'Hare International Airport, 94
distance between two moving planes, 112–113
height of a helicopter, 615
height of an airplane, 579, 588, 589
holding pattern, 470, 529
parking at O'Hare International Airport, 94
radar detection, 630
range of an airplane, A83

Archaeology

age of a fossil, 376
age of ancient tools, 371
date of prehistoric man's death, 390

Architecture

height of a statue on a building, 427
height of Eiffel Tower, 429
height of One World Trade Center, 430–431
Norman window, 185, A21
parabolic arch, 184
special window, 185
vertically circular buildings, 42

Area

bending a wire to enclose, 840
of Bermuda Triangle, A22
under a curve, 513
enclosing with a fence, 840
  cost of, 272
  maximizing, 184
of a lake, approximating, 615
of Norman window, maximizing, 185

Astronomy

angle of elevation of Sun, 577
Comet Hale-Bopp, 764–765
distance from Sun
  of Earth, 719
  of Jupiter, 719
  of Mars, 719
  of Mercury, 589, 863, 873
  of Pluto, 719
  of Venus, 590
distance to the Moon, 578, 588
Halley's Comet, 747
height of rock thrown on moon, 46, 48
limiting magnitude of a telescope, 390
orbits
  elliptical, 719
  of Mercury, 747
parallax method, 578

Aviation

orbital launches, 778
sonic boom, 732

Biology

bacterial population growth, 369, 376, 390–391
  of *E. coli*, 83
biorhythms, 470
blood pressure, 470
cricket chirping and temperature, 186
endangered species, 325
estimating age of a tree, 376
height and head circumference, 309
insect populations
  of fruit flies, 374–375
  growth of, 376
invasive species, 378
rabbit colony growth, 872
rats, humerus length in, 199
trout population, 872

Business

advertising, 35, 148, 199, 385
automobile production, 295
banquet seating, 855
break-even point, 139
can production, 270, 284
car production, 794
cost function, 140
costs
  average, 281
  of a can, 273
  of a commodity, 295
  computing, 822
  marginal, 173, 199
  minimizing, 173, 199, 860
  production, 83, 270, 295, 822, A13
  of transporting goods, 95
  variable, of manufacturing dishwashers, A81
depreciation, 890
  of machinery, 140, 390
  straight-line, 140
discounts, 296
expenses, computing, A82
inventory management, 116
juice production, 794
managing a meat market, 855–856
material needed to make a drum, 273
online purchases, 932
precision ball bearing manufacture, A13
production scheduling, 855
profit, 822
  computing, 822
  maximizing, 853–854, 855, 856
refunds, 779
revenues
  airlines, 856
  computing, 811–812
  daily, 173
  digital music, 110
  maximizing, 173, 179–181, 184
  monthly, 173
  price and, 178
  from sale of dishwashers, A81
  sales and, 178
  stadium seating, 891
  of Tesla, 383
  theaters, 780
sales
  cookie orders, 860
  markup of a new car, A91
  movie theater tickets, 767, 778
  net, 8
  presale orders, 779
  revenues and, 178
sales commissions, 198, A91
stocking a store, 935
transporting goods, 849
truck manufacture, 848
wages of a car salesperson, 34
watch production, A13

Calculus

difference quotient, 550
geometric series, 226
logistic growth model, 390
Mean Value Theorem, 84
Newton's Method, 258
Simpson's Rule, 186

Carpentry

pitch, 36

Chemistry

chemical reactions, 173
chlorine decomposition in a pool, 377
dinitrogen pentoxide decomposition, 377
Gas Laws, 121
mixing a solution, 779
mixing water and antifreeze, A82–A83
pH, 339
purity of gold, A83
salt decomposition in water, 377

salt solutions, A83
sucrose decomposition, 377
sugar molecules, A83
volume of a gas, A91

Combinatorics

airport codes, 915
baseball
  game outcomes, 935
  scheduling teams, 935
  teams, 922
basketball teams, 922
binary codes, 935
birthday permutations, 917, 922, 933, 935
blending coffees, A77–A78, A81
blending teas, A81
blouse and skirt combinations, 913
book arrangement, 922
choosing a team, 935
choosing seats on a bus, 935
code formation, 912, 915–916, 921
combination locks, 922
committee formation, 919, 921, 922, 935
flag arrangement, 920, 935
football teams, 922
license plate numbers, 922, 935
lining people up, 916, 921
mixing candy, A81
mixing nuts, 778, 849, 860, A81
multiple-choice test answers, 922
number formation, 913, 921
passwords, 922
selecting objects, 922
shirt and tie combinations, 913
stacking boxes, 921
stock names, 921
stocking a store and, 935
telephone numbers, 935
true/false test answers, 921
word forming, 920, 922

Communications

cellular phones
  cost of phone charges, 139
  data plans, 45, 127–128
  number of smartphone users, 377
fake news, 195
spreading of rumors, 325, 340
touch-tone phones, 566

Computers and computing

depreciation of, 140
graphics, 669, 823
number of households owning a tablet computer, 377
tablet plan, 94

Construction

of a box, 115
  closed, 125
  open, 159, 840
of a brick staircase, 880, 905
building an access ramp, 35
cement mix, A83
of a circular swimming pool, A21
of a citrus ladder, 880
of a coffee can, A83
of a cylindrical tube, 840
of enclosures
  maximizing area and, 184
  maximizing area of, 184
  of rectangular field, 184
of a flashlight, 708
of a headlight, 708
of a highway, 579, 589, 605, 615
of a loading ramp, 588
of a Norman window, 185, A21
pitch of a roof, 579
of a rain gutter, 185, 424–425, 561
of a road, 36, 430
of a stadium, 185, 880
of a TV dish, 708
weld inspection, 418
of a wooden deck, A82

Crime

forensics, 418
property crimes, 61
violent crimes, 61

Cryptography

forming codes, 912
matrices in, 822

Culture and society

fake news, 195
marital status of females, 914
marital status of males, 914
marriage problem, 341
social networking, 378, 384
spreading of rumors, 325, 340

Demographics

birth rate, 173
life cycle hypothesis, 185
living at home, 160
marital status of females, 914
marital status of males, 914

Design

of an awning, 589
of a ceramic floor, 878
of a closed box, 273
of decorative pieces, 430
Droste Effect, 873
of fasteners, 856
of a mosaic, 880, 905
of a racetrack, 718
of a searchlight, 708, 763
of websites, 823

Direction

of an aircraft, 663
of a swimmer, 696

Distance

across a pond, 429
of bicycle riding, 20
from Chicago to Honolulu, shortest, 514
height
  of an airplane, 579, 588, 589
  of the Bridge over the Royal Gorge, 588
  of Gibb's Hill Lighthouse, 569
  of Great Pyramid of Cheops, 589, A22
  of a helicopter, 615
  of a mountain, 587
  of Niagara Falls Incline Railway, 578
  of Willis Tower, 578
from Honolulu to Melbourne, Australia, shortest, 514
of hot-air balloon to intersection, 8
between John Hancock Center and Willis Tower, 578
to lamp shadow, 732
of lean of Leaning Tower of Pisa, 588
length of a ski lift, 587
to the Moon, 578, 588
to a plateau, 429
of a projectile, 442
range of an airplane, A83
of ranger from tower, 589
of rotating beacon, 479
at sea, 589
seen from submarine conning tower, A22
of a ship from shore, 429
to shore, 496
sound to measure, A74
between St. Louis and Oklahoma City, 588
stopping, 61, 173
from Sun
  Bode's Law for, 873
  to Earth, 719
  to Jupiter, 719
  to Mars, 719
  to Mercury, 589, 863, 873
  to Pluto, 719
  to Venus, 590
between two moving planes, 112–113
between two moving vehicles, 8, 114
between two objects, 429
between two runners, 588
width of Mississippi River, 578

Economics

consumer price index, 367
demand equation, 198–199
demand for candy, 120
economic mobility, 823
federal debt, per capita, 366
future value of money, 226
income taxes, 61, 84
  rate schedules for, 95
inflation, 366
IS-LM model, 779
labor force participation rate, 61
linear models in, 140, 199
national debt, 83–84
poverty rates, 225–226
poverty threshold, 9
profit function, 61
supply and demand, 136–137, 139

Education
college
 cost of, 366, 822
 funding, 390
 saving for, 890
 spring break, 855
 value of, 160
GPA and student work, 160
grade computation, A91
learning curve, 326, 340
Markov chains, 862
nonresident aliens earning doctorate degrees, 932
school loan interest, 822
video games and grade-point average, 147

Electricity
alternating (ac) current, 469, 496, 550
alternating (ac) current circuits, 488
alternating (ac) current generators, 469
current in an RC current, 326
current in an RL circuit, 326, 340
electrical engineering, 418
impedance and, A98
Kirchhoff's Rules, 780, 794
nuclear power plant, 731
parallel circuits, A98
rates in Florida, 35, 92
rates in Omaha, A91
resistance in a circuit, 120, A54, A57
 parallel circuits, 258
resistance of a wire, 121
solar energy, 20
voltage
 foreign, A13
 in United States, A13

Electronics
amplifier output, 390
microphones, 20
sawtooth curve on an oscilloscope, 561

Energy
conservation of, programming thermostats, 110
kinetic, 121
natural gas cost, 35, 94
nuclear power plant, 731
solar, 676, 708

Engineering
bridges
 arch, semielliptical, 763
 Golden Gate, 182
 parabolic arch, 199, 708
 semielliptical arch, 718
 suspension, 184, 708
crushing load, A74
electrical, 418
Gateway Arch, 708–709
lean of Leaning Tower of Pisa, 588
robotic arm, 686
weld instruction, 418

Entertainment
amphitheater seats, 879
Drury Lane Theater seats, 879
Ferris wheel, 470, 512, 529, 580, 590
fireworks display, 731
movie membership, 20
movie theater screens, 513
movie theater ticket sales, 767, 778
theater revenues, 780

Environment
cooling of air, 880
endangered species, 325
hours of daylight, 470, 486–487, 490, 512
invasive species, 378
length of day, 203, 286–287
oil leak, 295
pollutant in lake, 872
predator population, 455
relative humidity, 326
wood products, 375

Exercise and fitness
duration of exercise, A91
elliptical trainer, 719, 740

Finance
banking
 balancing a checkbook, A13
 comparing bank accounts, 366
 loans and, A81
college education
 cost of, 366, 822
 funding, 390
 saving for, 890
 value of, 160
cost
 of car rental, 95
 of college, 366, 822
 of driving a car, 34
 of electricity, 35
 of fencing, 272
 of first-class postage, 96
 fixed, 34
 of landscaping, 794
 of natural gas, 35, 94
 of parking at O'Hare International Airport, 94
 of phone charges, 139
 of production, 83
 of RV rental, 200
 of towing a car, 139
 of transatlantic travel, 61
 of transporting goods, 95
 of truck rental, 34
 variable, 34
cost equation, 34, 120
cost function, 140
credit cards
 balance owed on, 832
 debt, 872
 interest on, 366
 minimum payment for, 95
credit scores, 148
depreciation
 of machinery, 390
 straight-line, 135
financial planning, 779, 794, 795, 846, 848, 850, 856, A74–A75, A81
foreign exchange, 296
income
 annual earnings of young adults, 766, 862
 discretionary, 160
 life cycle hypothesis, 185
 salary and, 880, 890
 salary increases and, 905
interest
 compound, 359–360
 on credit cards, 366
 on a loan, A74
 on school loans, 822
loans, A81
 car, 872
 deciding among, 367
 interest on, A74
 paying off, 366
 school, interest on, 822
mortgages
 fees, 95
 interest rates, 366
 payments, 116–117, 120, 125
prices
 McDonald's, 48
 of U.S. postage stamps, 47
 of wine, 357
retirement savings, 890
revenue equation, 120
rich man's promise, 891
salary increases, 890, 905
saving
 for a car, 366
 for college, 890
 for a home, 890
 for retirement, 905
 savings plans, 366
 sinking fund and, 890
taxes
 federal withholding for, A91
 income, 139, 296, 309
 luxury, 139
value of a zero-coupon bond, 363

Food and nutrition
"light" foods, A91
animal nutrition, 856
blending coffees, 849, 860, A77–A78, A81
blending teas, A81
citrus ladders, 880
cookie orders, 860
cooling time of a pizza, 376
fast food
 cost of, 779
 prices of, 780
Girl Scout cookies, 932
ice cream, 856
juice production, 794

managing a meat market, 855–856
milk production, 384
mixing candy, A81
mixing nuts, 778, 849, 860, A81
number of servings, 780, 794
restaurant management, 779
time for a beer stein to warm, 377
wine and price, 357

Forestry

wood product classification, 375

Games

chess, 891
winning a lottery, 908, 933, 936–937

Geography

of Chicago road system, 629
distance across a pond, 429
distance to shore, 496
first to see rising Sun, 513
grade of a mountain trail, 615, 831, 841
inclination of a mountain trail, 575
land dimensions, 588
length of a lake, 496
Lewis and Clark expedition, 513
radioactive decay, 370–371, 376
tunnel clearance, 470
width of a gorge, 429
width of a river, 425–426, 496
width of Mississippi River, 578

Geology

earthquakes, 340, 763

Geometry

angles
  interior, of a polygon, 896
  between two lines, 551
  value of, 579
area, 181
  of an isosceles right triangle, 60, 419
  of an isosceles triangle, 561
  of an octagon, 560–561
  of a circle, 114
  cross-sectional, of a beam, 61, 69
  of a dodecagon, 551
  of a polygon, 806
  of a rectangle, 60, 114, 125, 159, 562, A12
  of a square, 114
  of a triangle, 120, 419, 806, A12
central angle of a circle, 419
circles
  central angle of, 419
  circumference of, 114, A12
  equation of, 806
  inscribing in a cone, 115
  inscribing in a sphere, 114
  radius of, 839
  tradius of, 420
circumference of a circle, 114, A12
colinear points, 806
cones
  volume of, 296
  volume of a right circular cone, 121
constant of proportionality, 120
cubes
  length of edge, 240
  volume of, A13
cylinders, volume of a right circular cylinder, 121, 296
diagonals, of rectangles, 199
dodecagon, area of, 551
equation of a circle, 806
equation of a line, 806
geometric mean, A91
hypocycloids and, 760
lemniscate, 21
length, of edge of a cube, 240
limaçon, 21
octagon, area of, 560–561
parallelepipeds, volume of, 686
perimeter
  of a rectangle, 114, 120, 839, A12
  of a square, 114
  of a triangle, A13
polygons
  area of, 806
  diagonals of, 160
  sum of interior angles of, 896
Pythagorean Theorem applications, A15
radius of circles, 839
rectangles
  area of, 125, 159, 562, A12
  diagonal of, 199
  formula for length and width of, 841
  perimeter of, 120, 839, A12
  vertices of, 199
right triangles
  angles of, 428
  hypotenuse, 577
  hypotenuse of, 428
sides of squares, 839
slope, 34
  of secant line, 83
spheres, 686
  surface area of, A13
  volume of, 120, A13
squares, sides of, 839
surface area
  of a balloon, 295
  of a sphere, A13
tetrahedrons, volume of, 806
triangles
  area of, 806, A12
  equilateral, 8, A12, A13
  isosceles, 8, 561, 839, 841
  medians, 7
  perimeter of, A13
unit circles, 419
vertices of rectangles, 199
volume
  of a balloon, 295
  of a cone, 115, 296
  of a cube, A13
  of a cylinder, 296
  of a parallelepiped, 686
  of a right circular cone, 121
  of a right circular cylinder, 121
  of a sphere, 120, A13
  of a tetrahedron, 806

Government

cost of first-class postage, 96
federal debt, 366
Senate committees, 922
taxes
  federal withholding for, A91
  income, 139, 296, 309
  luxury, 139

Health

age *versus* total cholesterol, 385
blood alcohol concentration, 335, 340–341
blood pressure, 529
body temperature, 195, A13
breast cancer, advanced-stage, 383
classifying blood types, 913
Covid-19, 383
expenditures on, 61
ideal body weight, 309
lung volume, 454
spread of a disease, 391
wound healing, 325, 339

Home improvement

installing a vent pipe, 718
painting a house, 780
painting a room, 480

Investment(s)

allocation of, 780
annuities
  amount of, 887–888
  tax-sheltered, 890
in bonds, zero-coupon, 363, 367
comparing
  of alternatives for, 366
  using different compounding periods, 360–361
IRA, comparing, 366
return on, 366, 855
in stock
  beta and, 129, 202
  companies listed on exchanges, 921
  portfolios, 914
  prices and, 891
  return on, 366
time to double or triple, 367
time to reach goals, 367

Landscaping

cost of, 794
enclosing a garden, A82
height of a tree, 588
moving a boulder, 669
pond enclosure, 199
stump removal, 668–669

Law and law enforcement

drivers stopped by police, by age, 392–393

Leisure and recreation
bungee jumping, 281
hot-air balloon, 429
length of a ski lift, 587
races
  distance between runners in a marathon, 588
  running in, 839
  between tortoise and hare, 839
RV rental cost, 200
spring break, 855

Measurement
of distance, using sound, A74
of height of CN Tower, 35
of height of Mt. Rushnore caricature, 429
of height of Washington Monument, 430
IQ tests, A91
of length of a guy wire, 35
of length of a lake, 496
optical, 538
parallax method, 578
of rainfall, 676
of stress of materials, 121
of temperature, 35, 110
of width of a river, 496

Medicine
ACL recovery, 441
concentration of a medication, 83
drug concentration, 272
lithotripsy, 719
maternal age and Down syndrome, 147
pancreatic cancer, advanced-stage, 325

Meteorology
height of a cloud, 426
rainfall measurement, 676
relative humidity, 326
wind chill factor, 391
wind speed computation, 779

Miscellaneous
board deflection, 747
carrying a ladder around a corner, 419, 479, 529–530
consecutive integers, 160
direction for crossing a river, 667
drafting error, 8
guy wires, 431
Old Faithful eruptions, 880
paper creases, 896
photography, 430, 441
reach of a ladder, 429
reading and interpreting graphs, 69–70
removing a discontinuity, 274
sewer bills, A91
waiting in line, 272
water leak, 747

Mixtures
cement mix, A83
mixing a solution, 779
mixing candy, A81
mixing drugs, 779, 795
mixing nuts, 778, 849, 860, A81
mixing water and antifreeze, A82–A83

Motion
projectile, 168–169, 172–173, 441, 763
simulating, 753–754
uniform, 114, 759, 763, A78–A80, A82

Motor vehicles
alcohol and driving, 335, 340–341
automobile production, 295
automobile theft and, 932
braking load, 676, 696
car loans, 872
car production, 794
crankshafts on cars, 589
depreciation of cars, 341, 357, 393
highway construction, 589
manufacturing trucks, 848
markup of a new car, A91
mixing water and antifreeze, A82–A83
piston engines, 429
runaway car, 178
saving for a car, 366
speed of a race car, 496
speed of a truck, 577
stopping distance, 309
weight of a car, 667

Music
digital, revenues from, 110
weight of a piano, 664

Navigation
bearing
  of an airplane, 576, 578
  of an object, 576
  of a ship, 578, 606, 615
charting a course, 667
distances at sea, 589
error correction, 606, 615
finding the correct compass heading, 667
light from a lighthouse, 575–576, A22
rescue at sea, 578, 585–586, 588
speed and direction of an aircraft, 663, 667
between St. Louis and Oklahoma City, 588
time lost to an error, 588

Oceanography
modeling tides, 470
tides, 490

Optics
bending light, 530
hyperbolic mirrors, 732
intensity of light, 121
lensmaker's equation, A57
light from a lighthouse, 575–576, A22
measurements, 538
percentage of light passing through a pane of glass, 324
photography, 430, 441
reflecting telescope, 708

Pets
animal nutrition, 856
ownership, 932

Pharmacy
drug concentration, 272, 325, 340
mixing drugs, 779, 795

Photography
security cameras, 578

Physics
amplifier output, 390
atmospheric pressure, 325, 339
bouncing balls, 890, 905
Brewster's Law, 530
cooling of a skillet, 390
Doppler effect, 273
elevation, weight and, 69
falling objects, 120
force
  braking load, 676, 696
  frictional, 578
  gravitational, 281
  incline angle, 676
  muscle, 668
  resultant, 667
  static equilibrium, 668, 669, 696
  of a truck pull, 668
  vectors of, 667
  of wind on a window, 118–119, 121
gravity
  acceleration due to, 258
  on Earth, 60, 309
  escape velocity and, 747
  force of, 281
  on Jupiter, 61
harmonic motion
  damped, 616
  simple, 616
heat loss
  through a wall, 118
  through a window, 125
heat transfer, 529
height of a ball, 185
height of rock thrown on moon, 46, 47, 48
Hooke's Law, 140
horsepower, 121
inclined plane, 428–429
inclined planes, A65
inclined ramp, 668
index of refraction, 530
intensity of light, 121
kinetic energy, 121
loudness of sound, 339
moment of inertia, 566
motion of a projectile, 168–169, 172–173, 441
Newton's Law, 120
  of Cooling, 371–373, 376
  of Heating, 377

object propelled vertically, 159–160, 178
pendulum motion, 110–111, 886, 890
  period of, 120, 309, A65, A74
product of inertia, 561
projectiles
  artillery, 178, 520
  distance, 442
  motion of, 529, 561, 566, 758–760
rate of change, instantaneous, 961, 964, 973
reflections in a mirror, 873
safe load for a beam, 121
speed of sound, 195
spring stiffness, 178
static friction, 668
stopping distance, 309
stress of materials, 121
stretching a spring, 120
tension in cables, 664–665, 668
velocity, instantaneous, 961–963, 964
vibrating string, 120
volume of a gas, 120
weight
  of a boat, 667
  of a car, 667
  elevation and, 69, 121, 125
  of a piano, 664
  supported by a piece of pine, 117–118

Population

bacterial, growth of, 369, 376, 390–391
birth rate, 173
of a city
  decline of, 376
  growth of, 376
of endangered species, 258, 377–378
as a function of age, 61
insect
  of fruit flies, 374–375
  growth of, 376, 378
  of mosquitoes, growth of, 376
population growth, 938, 975
Shannon's diversity index, 339
of a town, growth of, 378
of United States, 356–357, 384–385
of world, 357
  growth of, 863, 907
  predicting, 390

Probability

assigning, 931
of automobile theft, 932
choosing balls at random, 272
coin toss, 923
exponential, 325, 339–340
of Girl Scout cookie type, 932
of nonresident aliens earning doctorate degrees, 932
of online purchases, 932
of pet ownership, 932
Poisson, 325–326
of shared birthdays, 377
of unemployment, 935
of winning a lottery, 908, 933, 936–937
of YouTube usage, 932

Psychometrics

IQ tests, A91

Pyrotechnics

fireworks display, 731

Radioactivity

from Chernobyl, 377
decay, 370–371, 376, 383–384, 390, 856
Rutherford's experiment, 732

Real estate

apartment rents, 186
commissions, A91
cost of land, 615
housing prices, 284
  appreciation of, 366
mortgage fees, 95
valuing a home, 1, 44

Seismology

calibrating instruments, 763
magnitude of an earthquake, 340

Sequences

Fibonacci, 872
triangular numbers, 873

Speed

acceleration of spot of light from searchlight, 538
of an aircraft, 663
  radar detection of, 630
of the current of a stream, 779
of the current of Aguarico River, 860
of cyclists, A83
of lighthouse beacon rotation, 496
of a moving walkway, A82
of a sailboat, 645
of a swimmer, 696
of a truck, 577
walking, energy expended and, 65

Sports

baseball
  batting orders, 922
  diamond, 8
  games played to completion, 935
  Green Monster, 759
  home runs, 148
  Little League, 8
  scheduling games, 935
  team makeup, 922
  World Series, 922
basketball
  free-throw shots, 68–69, 579
  granny shots, 68
  team makeup, 922
bicycling
  distance of bicycle riding, 20
  speed of cyclists, A83
comparing Olympic heroes, A83
discus throw, 20
football
  seats in stadium, 879
  speed of players, A82
  stadium seating, 891
  team makeup, 922
  volume of a football, 718
golf, 385–386
  bunkers, 520
  motion of a golf ball, 69
pool shots, 430
races
  distance between two runners in a marathon, 588
  relay runner teams, 935
  running, A83
shot-put throw, 20
swimming, speed and direction of a swimmer, 696
tennis, 226, 273
  dimensions of court, A82

Surveys

analyzing data from, 910, 913

Technology

cellular phones
  cost of phone charges, 139
  data plans, 45, 127–128
  number of smartphone users, 377
constructing a TV dish, 708
directing a laser beam, 577
installing cable TV, 115
laser printers, A82
laser projection system, 561
mapping the mind, 500, 572
microphones, 20
recording devices, 732
robotic arm, 686
satellite dish, 705, 708
satellite receiver, 740
social networking, 378, 384
texting speed, 273
three-click rule, 823
touch-tone phones, 566
website maps, 823
YouTube usage, 932

Temperature

body temperature, 195, A13
conversion of, 140, 296, 309
cricket chirping and, 186
heat loss
  through a wall, 118
  through a window, 125
measurement of, 35, 110
modeling average monthly temperature, 470
monthly, 489–490
of a skillet, 390
time for a beer stein to warm, 377
wind chill, 95–96, 391

Tests and testing
IQ tests, A91

Time
for a beer stein to warm, 377
to double or triple an investment, 367
duration of exercise, A91
to empty a pool, A83
to empty a tub, A83
to empty an oil tanker, A83
to go from an island to a town, 115
hours of daylight, 470, 486–487, 490, 512
length of day, 203, 286–287
for a pizza to cool, 376
to reach a goal, 366
to reach an investment goal, 367
of a trip, 418–419, 430

Travel
catching a bus, 758
catching a train, 758
de-icing salt, 520
driving to school, 120
transatlantic, cost of, 61, 69

Volume
of a football, 718

Weapons
artillery, 178, 520

Weather
hurricanes, 147, 225, 489
lightning strikes, 728–729, 731
monthly temperature, 489–490, 496

Work
computing, 696
constant rate jobs, 860
moving a wagon, 674–675, 676
moving an object, 676, 686
student, GPA and, 160
unemployment and, 935
working together to do a job, A80, A82

Foundations: A Prelude to Functions

F

What Is My House Worth?

Credit: Rido/123RF.com

There are many factors that play a role in the value of a home. Everyone knows the golden rule of real estate—Location, Location, Location! Aside from where a property is located, one must consider the size of the home, number of bedrooms, number of bathrooms, status of updates within the home, and many, many other considerations.

Zillow (www.zillow.com) developed a model (an equation) that is used to approximate the value of a home. This approximate value is called a Zestimate. According to Zillow, the Zestimate is the estimated market value for an individual home. Zillow uses available information on the millions of homes that have sold around the country to arrive at its Zestimate. Mainly, Zillow uses the physical attributes of the home, tax assessments, and transaction data to arrive at its Zestimate. Homeowners are free to report updated home facts about their particular property in order to improve the Zestimate. The accuracy of the Zestimate is dependent upon the location of the home. For example, Zestimates in the Chicago area have some of the best Zestimates (60.9% of homes sold within 5% of the Zestimate), while Zestimates in Cleveland, OH, are only fair (44.2% of the homes sold within 5% of the sale price).

—Michael Sullivan, III

***Source**: https://www.zillow.com*

—See the Internet-based Chapter Project—

A Look Back

Appendix A reviews skills from Intermediate Algebra.

A Look Ahead

Here we connect algebra and geometry using the rectangular coordinate system. In the 1600s, algebra had developed to the point that René Descartes (1596–1650) and Pierre de Fermat (1601–1665) were able to use rectangular coordinates to translate geometry problems into algebra problems, and vice versa. This enabled both geometers and algebraists to gain new insights into their subjects, which had been thought to be separate but now were seen as connected.

Outline

F.1 The Distance and Midpoint Formulas
F.2 Graphs of Equations in Two Variables; Intercepts; Symmetry
F.3 Lines
F.4 Circles
Chapter Project

F.1 The Distance and Midpoint Formulas

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Algebra Essentials (Appendix A, Section A.1, pp. A1–A10)
- Geometry Essentials (Appendix A, Section A.2, pp. A14–A19)

Now Work the **'Are You Prepared?'** problems on page 6.

OBJECTIVES **1** Use the Distance Formula (p. 3)
2 Use the Midpoint Formula (p. 5)

Rectangular Coordinates

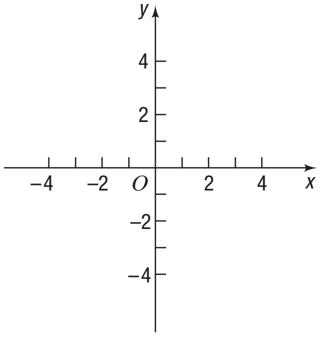


Figure 1 *xy*-Plane

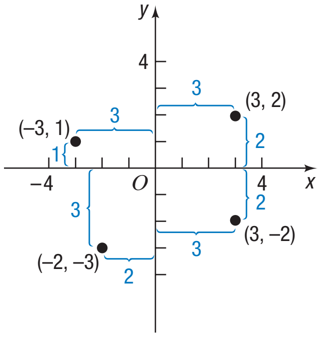


Figure 2

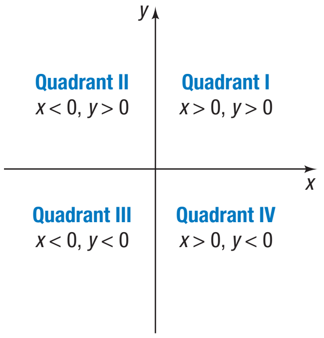


Figure 3

We locate a point on the real number line by assigning it a single real number, called the *coordinate of the point*. For work in a two-dimensional plane, we locate points by using two numbers.

Begin with two real number lines located in the same plane: one horizontal and the other vertical. The horizontal line is called the ***x*-axis**, the vertical line the ***y*-axis**, and the point of intersection the **origin *O***. See Figure 1. Assign coordinates to every point on these number lines using a convenient scale. In mathematics, we usually use the same scale on each axis, but in applications, different scales appropriate to the application may be used.

The origin O has a value of 0 on both the x-axis and the y-axis. Points on the x-axis to the right of O are associated with positive real numbers, and those to the left of O are associated with negative real numbers. Points on the y-axis above O are associated with positive real numbers, and those below O are associated with negative real numbers. In Figure 1, the x-axis and y-axis are labeled as x and y, respectively, and an arrow at the end of each axis is used to denote the positive direction.

The coordinate system described here is called a **rectangular** or **Cartesian*** **coordinate system**. The x-axis and y-axis lie in a *plane* called the ***xy*-plane**, and the x-axis and y-axis are referred to as the **coordinate axes**.

Any point P in the xy-plane can be located by using an **ordered pair** (x, y) of real numbers. Let x denote the signed distance of P from the y-axis (*signed* means that if P is to the right of the y-axis, then $x > 0$, and if P is to the left of the y-axis, then $x < 0$); and let y denote the signed distance of P from the x-axis. The ordered pair (x, y), also called the **coordinates** of P, gives us enough information to locate the point P in the plane.

For example, to locate the point whose coordinates are $(-3, 1)$, go 3 units along the x-axis to the left of O and then go straight up 1 unit. We **plot** this point by placing a dot at this location. See Figure 2, in which the points with coordinates $(-3, 1)$, $(-2, -3)$, $(3, -2)$, and $(3, 2)$ are plotted.

The origin has coordinates $(0, 0)$. Any point on the x-axis has coordinates of the form $(x, 0)$, and any point on the y-axis has coordinates of the form $(0, y)$.

If (x, y) are the coordinates of a point P, then x is called the ***x*-coordinate**, or **abscissa**, of P, and y is the ***y*-coordinate**, or **ordinate**, of P. We identify the point P by its coordinates (x, y) by writing $P = (x, y)$. Usually, we will simply say "the point (x, y)" rather than "the point whose coordinates are (x, y)."

The coordinate axes partition the xy-plane into four sections called **quadrants**, as shown in Figure 3. In quadrant I, both the x-coordinate and the y-coordinate of all points are positive; in quadrant II, x is negative and y is positive; in quadrant III, both x and y are negative; and in quadrant IV, x is positive and y is negative. Points on the coordinate axes belong to no quadrant.

Now Work PROBLEM 15

*Named after René Descartes (1596–1650), a French mathematician, philosopher, and theologian.

COMMENT With a graphing utility, you can set the scale on each axis. Once this has been done, you obtain the **viewing rectangle**. See Figure 4 for a typical viewing rectangle. You should now read Section B.1, *The Viewing Rectangle*, in Appendix B.

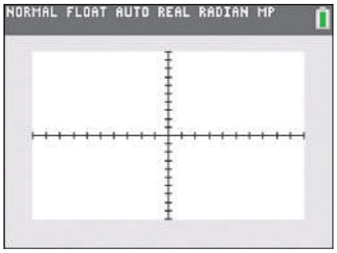


Figure 4 TI-84 Plus CE Standard Viewing Rectangle

1 Use the Distance Formula

If the same units of measurement (such as inches, centimeters, and so on) are used for both the x-axis and y-axis, then all distances in the xy-plane can be measured using this unit of measurement.

EXAMPLE 1 Finding the Distance Between Two Points

Find the distance d between the points $(1, 3)$ and $(5, 6)$.

Solution

First plot the points $(1, 3)$ and $(5, 6)$ and connect them with a line segment. See Figure 5(a). To find the length d, begin by drawing a horizontal line segment from $(1, 3)$ to $(5, 3)$ and a vertical line segment from $(5, 3)$ to $(5, 6)$, forming a right triangle, as shown in Figure 5(b). One leg of the triangle is of length 4 (since $|5 - 1| = 4$), and the other is of length 3 (since $|6 - 3| = 3$). By the Pythagorean Theorem, the square of the distance d that we seek is

Need to Review?
The Pythagorean Theorem and its converse are discussed in Section A.2, pp. A14–A15.

$$d^2 = 4^2 + 3^2 = 16 + 9 = 25$$
$$d = \sqrt{25} = 5$$

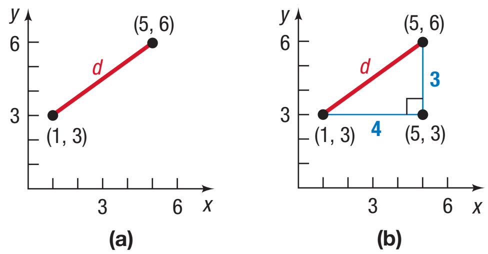


Figure 5

The **distance formula** provides a straightforward method for computing the distance between two points.

THEOREM Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \textbf{(1)}$$

In Words
To compute the distance between two points, find the difference of the x-coordinates, square it, and add this to the square of the difference of the y-coordinates. The square root of this sum is the distance.

Proof of the Distance Formula Let (x_1, y_1) denote the coordinates of point P_1 and let (x_2, y_2) denote the coordinates of point P_2.

- Assume that the line joining P_1 and P_2 is neither horizontal nor vertical. Refer to Figure 6(a) on the next page. The coordinates of P_3 are (x_2, y_1). The horizontal

(*continued*)

distance from P_1 to P_3 equals the absolute value of the difference of the x-coordinates, $|x_2 - x_1|$. The vertical distance from P_3 to P_2 equals the absolute value of the difference of the y-coordinates, $|y_2 - y_1|$. See Figure 6(b). The distance $d(P_1, P_2)$ is the length of the hypotenuse of the right triangle, so, by the Pythagorean Theorem, it follows that

$$\begin{aligned}[d(P_1, P_2)]^2 &= |x_2 - x_1|^2 + |y_2 - y_1|^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\end{aligned}$$

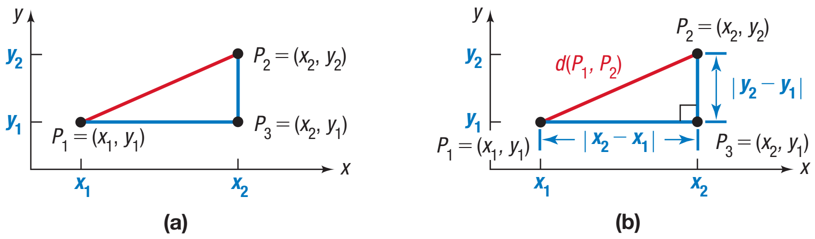


Figure 6

- If the line joining P_1 and P_2 is horizontal, then the y-coordinate of P_1 equals the y-coordinate of P_2; that is, $y_1 = y_2$. Refer to Figure 7(a). In this case, the distance formula (1) still works, because for $y_1 = y_2$, it reduces to

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + 0^2} = \sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$$

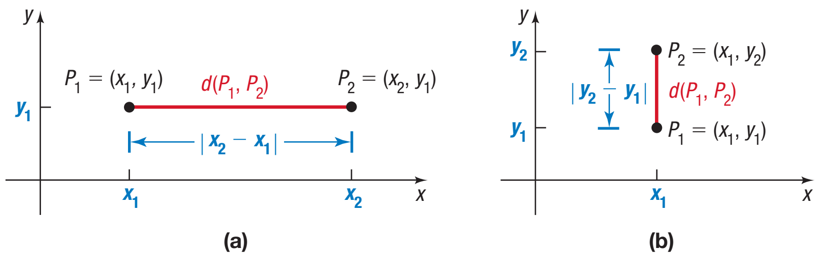


Figure 7

- A similar argument holds if the line joining P_1 and P_2 is vertical. See Figure 7(b).

■

EXAMPLE 2 Using the Distance Formula

Find the distance d between the points $(-4, 5)$ and $(3, 2)$.

Solution Using the distance formula (1) reveals that the distance d is

$$\begin{aligned}d &= \sqrt{[3 - (-4)]^2 + (2 - 5)^2} = \sqrt{7^2 + (-3)^2} \\ &= \sqrt{49 + 9} = \sqrt{58} \approx 7.62\end{aligned}$$

Now Work PROBLEMS 19 AND 23

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is never a negative number. Also, the distance between two points is 0 only when the points are identical—that is, when $x_1 = x_2$ and $y_1 = y_2$. And, because $(x_2 - x_1)^2 = (x_1 - x_2)^2$ and $(y_2 - y_1)^2 = (y_1 - y_2)^2$, it makes no difference whether the distance is computed from P_1 to P_2 or from P_2 to P_1; that is, $d(P_1, P_2) = d(P_2, P_1)$.

The introduction to this chapter mentioned that rectangular coordinates enable us to translate geometry problems into algebra problems, and vice versa. The next example shows how algebra (the distance formula) can be used to solve geometry problems.

EXAMPLE 3 Using Algebra to Solve a Geometry Problem

Consider the three points $A = (-2, 1)$, $B = (2, 3)$, and $C = (3, 1)$.

(a) Plot each point and form the triangle ABC.
(b) Find the length of each side of the triangle.
(c) Show that the triangle is a right triangle.
(d) Find the area of the triangle.

Solution

(a) Figure 8 shows the points A, B, C and the triangle ABC.

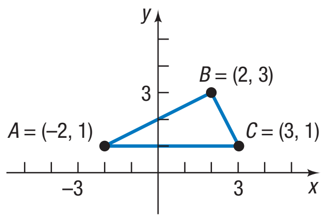


Figure 8

(b) To find the length of each side of the triangle, use the distance formula, equation (1).

$$d(A, B) = \sqrt{[2-(-2)]^2 + (3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$
$$d(B, C) = \sqrt{(3-2)^2 + (1-3)^2} = \sqrt{1+4} = \sqrt{5}$$
$$d(A, C) = \sqrt{[3-(-2)]^2 + (1-1)^2} = \sqrt{25+0} = 5$$

(c) If the sum of the squares of the lengths of two of the sides equals the square of the length of the third side, then the triangle is a right triangle. Looking at Figure 8, it seems reasonable to conjecture that the angle at vertex B might be a right angle. We shall check to see whether

$$[d(A, B)]^2 + [d(B, C)]^2 = [d(A, C)]^2$$

Using the results in part (b) yields

$$[d(A, B)]^2 + [d(B, C)]^2 = \left(2\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2 = 20 + 5 = 25 = [d(A, C)]^2$$

It follows from the converse of the Pythagorean Theorem that triangle ABC is a right triangle.

(d) Because the right angle is at vertex B, the sides AB and BC form the base and height of the triangle. Its area is

$$\text{Area} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height} = \frac{1}{2} \cdot 2\sqrt{5} \cdot \sqrt{5} = 5 \text{ square units}$$

Now Work PROBLEM 33

2 Use the Midpoint Formula

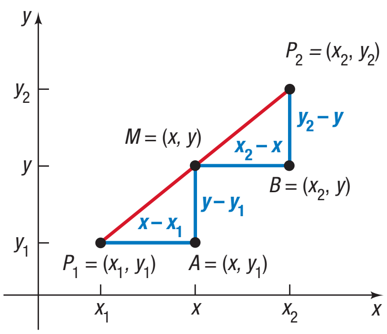


Figure 9

We now derive a formula for the coordinates of the **midpoint of a line segment**. Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be the endpoints of a line segment, and let $M = (x, y)$ be the point on the line segment that is the same distance from P_1 as it is from P_2. See Figure 9. The triangles P_1AM and MBP_2 are congruent. [Do you see why? $d(P_1, M) = d(M, P_2)$ is given; also, $\angle AP_1M = \angle BMP_2$* and $\angle P_1MA = \angle MP_2B$. So, we have Angle–Side–Angle.] Because triangles P_1AM and MBP_2 are congruent, corresponding sides are equal in length. That is,

$$x - x_1 = x_2 - x \qquad \text{and} \qquad y - y_1 = y_2 - y$$
$$2x = x_1 + x_2 \qquad\qquad 2y = y_1 + y_2$$
$$x = \frac{x_1 + x_2}{2} \qquad\qquad y = \frac{y_1 + y_2}{2}$$

*A postulate from geometry states that the transversal $\overline{P_1P_2}$ forms congruent corresponding angles with the parallel line segments $\overline{P_1A}$ and $\overline{MB}$.

In Words
To find the midpoint of a line segment, average the x-coordinates of the endpoints, and average the y-coordinates of the endpoints.

THEOREM Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \textbf{(2)}$$

EXAMPLE 4 Finding the Midpoint of a Line Segment

Find the midpoint of the line segment from $P_1 = (-5, 5)$ to $P_2 = (3, 1)$. Plot the points P_1 and P_2 and their midpoint.

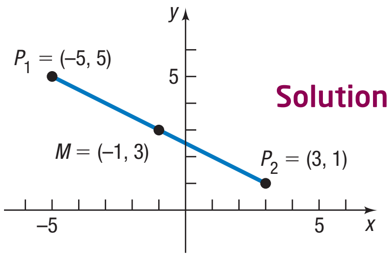


Figure 10

Solution Use the midpoint formula (2) with $x_1 = -5$, $y_1 = 5$, $x_2 = 3$, and $y_2 = 1$. The coordinates (x, y) of the midpoint M are

$$x = \frac{x_1 + x_2}{2} = \frac{-5 + 3}{2} = -1 \quad \text{and} \quad y = \frac{y_1 + y_2}{2} = \frac{5 + 1}{2} = 3$$

That is, $M = (-1, 3)$. See Figure 10.

Now Work PROBLEM 39

F.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. On the real number line, the origin is assigned the number _______. (p. A4)
2. If –3 and 5 are the coordinates of two points on the real number line, the distance between these points is _______. (pp. A5–A6)
3. If 3 and 4 are the legs of a right triangle, the hypotenuse is _______. (p. A14)
4. Use the converse of the Pythagorean Theorem to show that a triangle whose sides are of lengths 11, 60, and 61 is a right triangle. (pp. A14–A15)
5. The area A of a triangle whose base is b and whose altitude is h is $A =$ _______________. (p. A15)
6. ***True or False*** Two triangles are congruent if two angles and the included side of one equals two angles and the included side of the other. (pp. A16–A17)

Concepts and Vocabulary

7. If (x, y) are the coordinates of a point P in the xy-plane, then x is called the ____________________ of P, and y is the ____________________ of P.
8. The coordinate axes partition the xy-plane into four sections called __________.
9. If three distinct points P, Q, and R all lie on a line, and if $d(P, Q) = d(Q, R)$, then Q is called the ________ of the line segment from P to R.
10. ***True or False*** The distance between two points is sometimes a negative number.
11. ***True or False*** The point $(-1, 4)$ lies in quadrant IV of the Cartesian plane.
12. ***True or False*** The midpoint of a line segment is found by averaging the x-coordinates and averaging the y-coordinates of the endpoints.
13. ***Multiple Choice*** Which of the following statements is true for a point (x, y) that lies in quadrant III?
    (a) Both x and y are positive.
    (b) Both x and y are negative.
    (c) x is positive, and y is negative.
    (d) x is negative, and y is positive.
14. ***Multiple Choice*** Choose the expression that equals the distance between two points (x_1, y_1) and (x_2, y_2).
    (a) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
    (b) $\sqrt{(x_2 + x_1)^2 - (y_2 + y_1)^2}$
    (c) $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$
    (d) $\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$

Skill Building

In Problems 15 and 16, plot each point in the xy-plane. State which quadrant or on what coordinate axis each point lies.

15. **(a)** $A = (-3, 2)$ **(b)** $B = (6, 0)$ **(c)** $C = (-2, -2)$ **(d)** $D = (6, 5)$ **(e)** $E = (0, -3)$ **(f)** $F = (6, -3)$

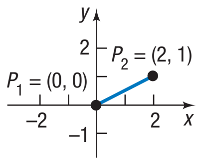
16. **(a)** $A = (1, 4)$ **(b)** $B = (-3, -4)$ **(c)** $C = (-3, 4)$ **(d)** $D = (4, 1)$ **(e)** $E = (0, 1)$ **(f)** $F = (-3, 0)$

17. Plot the points $(2, 0)$, $(2, -3)$, $(2, 4)$, $(2, 1)$, and $(2, -1)$. Describe the set of all points of the form $(2, y)$, where y is a real number.

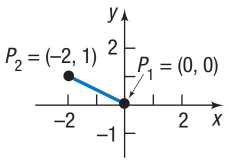
18. Plot the points $(0, 3)$, $(1, 3)$, $(-2, 3)$, $(5, 3)$, and $(-4, 3)$. Describe the set of all points of the form $(x, 3)$, where x is a real number.

In Problems 19–32, find the distance d between the points P_1 and P_2.

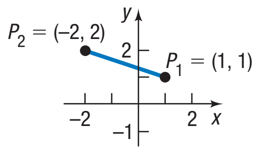
19.



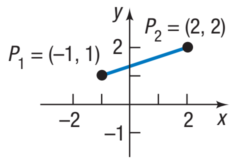
20.



21.



22.



23. $P_1 = (3, -4)$; $P_2 = (5, 4)$

24. $P_1 = (-1, 0)$; $P_2 = (2, 4)$

25. $P_1 = (-7, 3)$; $P_2 = (4, 0)$

26. $P_1 = (2, -3)$; $P_2 = (4, 2)$

27. $P_1 = (5, -2)$; $P_2 = (6, 1)$

28. $P_1 = (-4, -3)$; $P_2 = (6, 2)$

29. $P_1 = (-0.2, 0.3)$; $P_2 = (2.3, 1.1)$

30. $P_1 = (1.2, 2.3)$; $P_2 = (-0.3, 1.1)$

31. $P_1 = (a, b)$; $P_2 = (0, 0)$

32. $P_1 = (a, a)$; $P_2 = (0, 0)$

In Problems 33–38, plot each point and form the triangle ABC. Show that the triangle is a right triangle. Find its area.

33. $A = (-2, 5)$; $B = (1, 3)$; $C = (-1, 0)$

34. $A = (-2, 5)$; $B = (12, 3)$; $C = (10, -11)$

35. $A = (-5, 3)$; $B = (6, 0)$; $C = (5, 5)$

36. $A = (-6, 3)$; $B = (3, -5)$; $C = (-1, 5)$

37. $A = (4, -3)$; $B = (0, -3)$; $C = (4, 2)$

38. $A = (4, -3)$; $B = (4, 1)$; $C = (2, 1)$

In Problems 39–46, find the midpoint of the line segment joining the points P_1 and P_2.

39. $P_1 = (3, -4)$; $P_2 = (5, 4)$

40. $P_1 = (-2, 0)$; $P_2 = (2, 4)$

41. $P_1 = (-1, 4)$; $P_2 = (8, 0)$

42. $P_1 = (2, -3)$; $P_2 = (4, 2)$

43. $P_1 = (7, -5)$; $P_2 = (9, 1)$

44. $P_1 = (-4, -3)$; $P_2 = (2, 2)$

45. $P_1 = (a, b)$; $P_2 = (0, 0)$

46. $P_1 = (a, a)$; $P_2 = (0, 0)$

Applications and Extensions

47. If the point $(2, 5)$ is shifted 3 units to the right and 2 units down, what are its new coordinates?

48. If the point $(-1, 6)$ is shifted 2 units to the left and 4 units up, what are its new coordinates?

49. Find all points having an x-coordinate of 3 whose distance from the point $(-2, -1)$ is 13.
(a) By using the Pythagorean Theorem.
(b) By using the distance formula.

50. Find all points having a y-coordinate of -6 whose distance from the point $(1, 2)$ is 17.
(a) By using the Pythagorean Theorem.
(b) By using the distance formula.

51. Find all points on the x-axis that are 6 units from the point $(4, -3)$.

52. Find all points on the y-axis that are 6 units from the point $(4, -3)$.

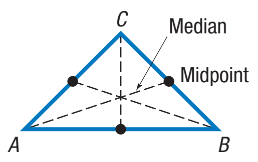
53. Suppose that $A = (2, 5)$ are the coordinates of a point in the xy-plane.
(a) Find the coordinates of the point if A is shifted 3 units to the left and 4 units down.
(b) Find the coordinates of the point if A is shifted 2 units to the left and 8 units up.

54. Plot the points $A = (-1, 8)$ and $M = (2, 3)$ in the xy-plane. If M is the midpoint of a line segment AB, find the coordinates of B.

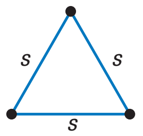
55. The midpoint of the line segment from P_1 to P_2 is $(-1, 4)$. If $P_1 = (-3, 6)$, what is P_2?

56. The midpoint of the line segment from P_1 to P_2 is $(5, -4)$. If $P_2 = (7, -2)$, what is P_1?

57. **Geometry** The **medians** of a triangle are the line segments from each vertex to the midpoint of the opposite side (see the figure). Find the lengths of the medians of the triangle with vertices at $A = (0, 0)$, $B = (6, 0)$, and $C = (4, 4)$.



58. Geometry An **equilateral triangle** has three sides of equal length. If two vertices of an equilateral triangle are $(0, 4)$ and $(0, 0)$ find the third vertex. How many of these triangles are possible?



*In Problems 59–62, find the length of each side of the triangle determined by the three points P_1, P_2, and P_3. State whether the triangle is an isosceles triangle, a right triangle, neither of these, or both. (An **isosceles triangle** is one in which at least two of the sides are of equal length.)*

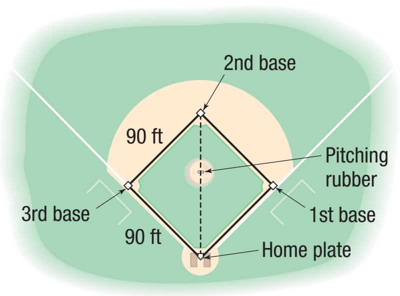
59. $P_1 = (2, 1);\quad P_2 = (-4, 1);\quad P_3 = (-4, -3)$

60. $P_1 = (-1, 4);\quad P_2 = (6, 2);\quad P_3 = (4, -5)$

61. $P_1 = (-2, -1);\quad P_2 = (0, 7);\quad P_3 = (3, 2)$

62. $P_1 = (7, 2);\quad P_2 = (-4, 0);\quad P_3 = (4, 6)$

63. Baseball A major league baseball "diamond" is actually a square 90 feet on a side (see the figure). What is the distance directly from home plate to second base (the diagonal of the square)?



64. Little League Baseball The layout of a Little League playing field is a square 60 feet on a side. How far is it directly from home plate to second base (the diagonal of the square)?

Source: *2022 Little League Baseball Official Regulations, Playing Rules, and Operating Policies*

65. Baseball Refer to Problem 63. Overlay a rectangular coordinate system on a major league baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.

(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.

(b) If the right fielder is located at $(310, 15)$ how far is it from the right fielder to second base?

(c) If the center fielder is located at $(300, 300)$, how far is it from the center fielder to third base?

66. Little League Baseball Refer to Problem 64. Overlay a rectangular coordinate system on a Little League baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.

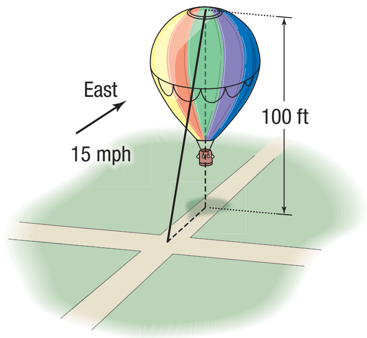
(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.

(b) If the right fielder is located at $(180, 20)$, how far is it from the right fielder to second base?

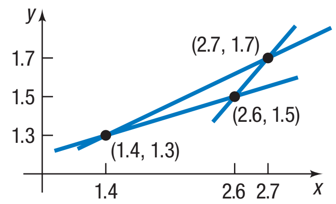
(c) If the center fielder is located at $(220, 220)$, how far is it from the center fielder to third base?

67. Distance Between Moving Objects A Ford Focus car and a Freightliner Cascadia truck leave an intersection at the same time. The Focus heads east at an average speed of 60 miles per hour, while the Cascadia heads south at an average speed of 45 miles per hour. Find an expression for their distance apart d (in miles) at the end of t hours.

68. Distance of a Moving Object from a Fixed Point A hot-air balloon, headed due east at an average speed of 15 miles per hour and at a constant altitude of 100 feet, passes over an intersection (see the figure). Find an expression for the distance d (measured in feet) from the balloon to the intersection t seconds later.



69. Drafting Error When a drafter draws three lines that are to intersect at one point, the lines may not intersect as intended and subsequently will form an **error triangle**. If this error triangle is long and thin, one estimate for the location of the desired point is the midpoint of the shortest side. The figure shows one such error triangle.

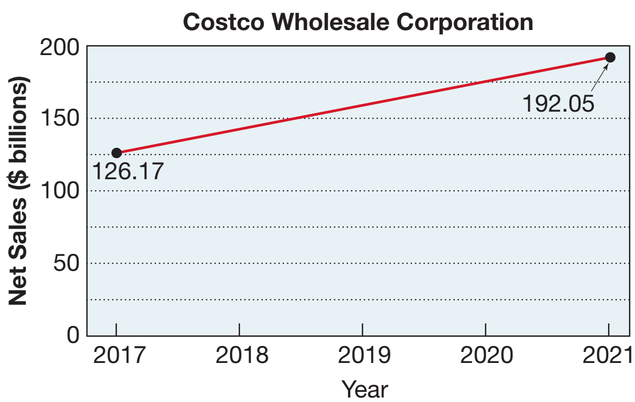


(a) Find an estimate for the desired intersection point.

(b) Find the distance from (1.4, 1.3) to the midpoint found in part (a).

70. Net Sales The figure illustrates the net sales growth of Costco Wholesale Corporation from 2017 through 2021. Use the midpoint formula to estimate the net sales of Costco Wholesale Corporation in 2019. How does your result compare to the reported value of $149.35 billion?

Source: *Costco Wholesale Corporation 2021 Annual Report*



71. **Poverty Threshold** Poverty thresholds are determined by the U.S. Census Bureau. A poverty threshold represents the minimum annual household income for a family not to be considered poor. In 2013, the poverty threshold for a family of four with two children under the age of 18 years was $23,624. In 2021, the poverty threshold for a family of four with two children under the age of 18 years was $27,479. Assuming that poverty thresholds increase in a straight-line fashion, use the midpoint formula to estimate the poverty threshold for a family of four with two children under the age of 18 in 2017. How does your result compare to the actual poverty threshold in 2017 of $24,858?

Source: *U.S. Census Bureau*

72. ***Challenge Problem*** **Geometry** Verify that the points $(0, 0)$, $(a, 0)$, and $\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$ are the vertices of an equilateral triangle. Then show that the midpoints of the three sides are the vertices of a second equilateral triangle.

73. ***Challenge Problem*** **Geometry** Find the midpoint of each diagonal of a square with side of length s. Draw the conclusion that the diagonals of a square intersect at their midpoints.

[**Hint**: Use $(0, 0)$, $(0, s)$, $(s, 0)$, and (s, s) as the vertices of the square.]

74. ***Challenge Problem*** **Geometry** A point P is equidistant from $(-5, 1)$ and $(4, -4)$. Find the coordinates of P if its y-coordinate is twice its x-coordinate.

75. ***Challenge Problem*** **Geometry** For any parallelogram, prove that the sum of the squares of the lengths of the sides equals the sum of the squares of the lengths of the diagonals.

[**Hint**: Use $(0, 0)$, $(a, 0)$, $(a + b, c)$, and (b, c) as the vertices of the parallelogram.]

Explaining Concepts: Discussion and Writing

76. Write a paragraph that describes a Cartesian plane. Then write a second paragraph that describes how to plot points in the Cartesian plane. Your paragraphs should include the terms *coordinate axes, ordered pair, coordinates, plot, x-coordinate,* and *y-coordinate.*

'Are You Prepared?' Answers

1. 0 **2.** 8 **3.** 5 **4.** $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$ **5.** $\frac{1}{2}bh$ **6.** True

F.2 Graphs of Equations in Two Variables; Intercepts; Symmetry

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Solving Equations (Appendix A, Section A.8, pp. A66–A72)

Now Work the **'Are You Prepared?'** problems on page 18.

OBJECTIVES
1. Graph Equations by Plotting Points (p. 9)
2. Find Intercepts from a Graph (p. 12)
3. Find Intercepts from an Equation (p. 13)
4. Test an Equation for Symmetry with Respect to the x-Axis, the y-Axis, and the Origin (p. 13)
5. Know How to Graph Key Equations (p. 16)

1 Graph Equations by Plotting Points

An **equation in two variables**, say x and y, is a statement in which two expressions involving x and y are equal. The expressions are called the **sides** of the equation. Since an equation is a statement, it may be true or false, depending on the value of the variables. Any values of x and y that result in a true statement are said to **satisfy** the equation.

For example, the following are all equations in two variables x and y:

$$x^2 + y^2 = 5 \qquad 2x - y = 6 \qquad y = 2x + 5 \qquad x^2 = y$$