COLLEGE ALGEBRA

10E

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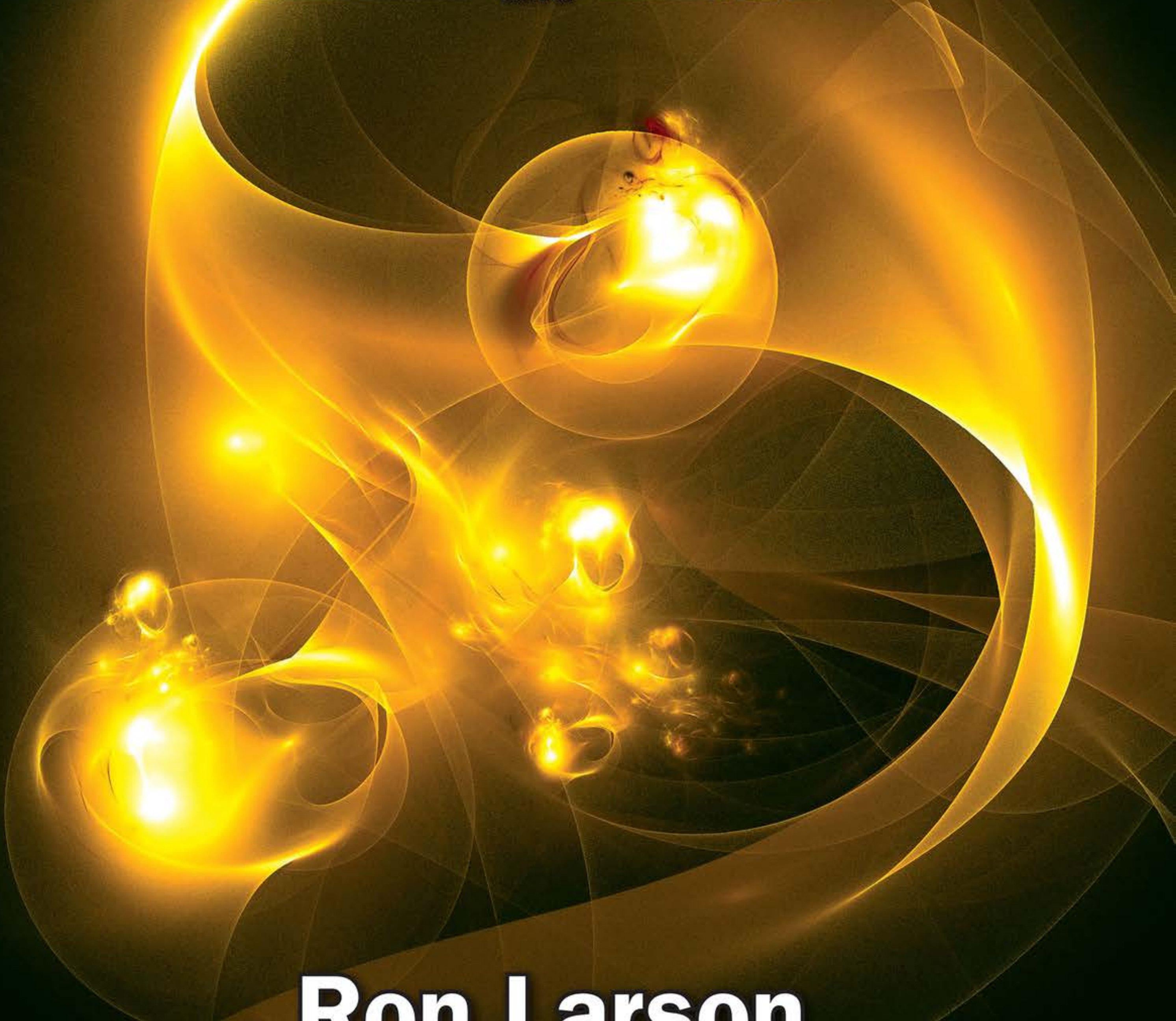

Ron Larson

COLLEGE ALGEBRA

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CalcChat® and CalcView®

10E

Ron Larson
The Pennsylvania State University
The Behrend College

With the assistance of David C. Falvo
The Pennsylvania State University
The Behrend College

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College Algebra
with CalcChat and CalcView
Tenth Edition

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Contents

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Preface

Welcome to *College Algebra,* Tenth Edition. We are excited to offer you a new edition with even more resources that will help you understand and master algebra. This textbook includes features and resources that continue to make *College Algebra* a valuable learning tool for students and a trustworthy teaching tool for instructors.

College Algebra provides the clear instruction, precise mathematics, and thorough coverage that you expect for your course. Additionally, this new edition provides you with **free** access to three companion websites:

- **CalcView.com**—video solutions to selected exercises
- **CalcChat.com**—worked-out solutions to odd-numbered exercises and access to online tutors
- **LarsonPrecalculus.com**—companion website with resources to supplement your learning

These websites will help enhance and reinforce your understanding of the material presented in this text and prepare you for future mathematics courses. CalcView® and CalcChat® are also available as free mobile apps.

Features

NEW CalcView®

The website *CalcView.com* contains video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google Play™ store. The app features an embedded QR Code® reader that can be used to scan the on-page codes and go directly to the videos. You can also access the videos at *CalcView.com.*

UPDATED CalcChat®

In each exercise set, be sure to notice the reference to *CalcChat.com.* This website provides free step-by-step solutions to all odd-numbered exercises in many of our textbooks. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. For over 14 years, hundreds of thousands of students have visited this site for help. The CalcChat mobile app is also available as a free download at the Apple® App Store® or Google Play™ store and features an embedded QR Code® reader.

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REVISED LarsonPrecalculus.com

All companion website features have been updated based on this revision, plus we have added a new Collaborative Project feature. Access to these features is free. You can view and listen to worked-out solutions of Checkpoint problems in English or Spanish, explore examples, download data sets, watch lesson videos, and much more.

NEW Collaborative Project

You can find these extended group projects at *LarsonPrecalculus.com.* Check your understanding of the chapter concepts by solving in-depth, real-life problems. These collaborative projects provide an interesting and engaging way for you and other students to work together and investigate ideas.

REVISED Exercise Sets

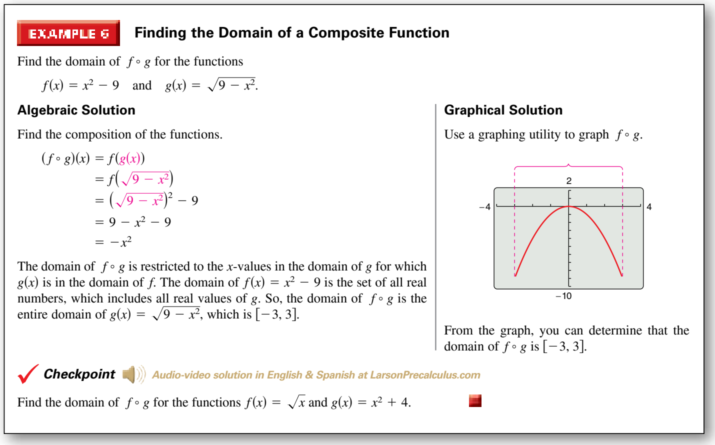
The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant, and include topics our users have suggested. The exercises have been reorganized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations. Error Analysis exercises have been added throughout the text to help you identify common mistakes.

Chapter Opener

Each Chapter Opener highlights real-life applications used in the examples and exercises.

Section Objectives

A bulleted list of learning objectives provides you the opportunity to preview what will be presented in the upcoming section.



EXAMPLE 6 Finding the Domain of a Composite Function

Find the domain of $f \circ g$ for the functions

$f(x) = x^2 - 9$ and $g(x) = \sqrt{9 - x^2}$.

Algebraic Solution

Find the composition of the functions.

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{9-x^2}) = (\sqrt{9-x^2})^2 - 9 = 9 - x^2 - 9 = -x^2$$

The domain of $f \circ g$ is restricted to the x-values in the domain of g for which $g(x)$ is in the domain of f. The domain of $f(x) = x^2 - 9$ is the set of all real numbers, which includes all real values of g. So, the domain of $f \circ g$ is the entire domain of $g(x) = \sqrt{9 - x^2}$, which is $[-3, 3]$.

Graphical Solution

Use a graphing utility to graph $f \circ g$.

From the graph, you can determine that the domain of $f \circ g$ is $[-3, 3]$.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Find the domain of $f \circ g$ for the functions $f(x) = \sqrt{x}$ and $g(x) = x^2 + 4$.

Side-By-Side Examples

Throughout the text, we present solutions to many examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.

Remarks

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.

Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at *LarsonPrecalculus.com.*

Technology

The technology feature gives suggestions for effectively using tools such as calculators, graphing utilities, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

Historical Notes

These notes provide helpful information regarding famous mathematicians and their work.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol ƒ.

Summarize

The Summarize feature at the end of each section helps you organize the lesson's key concepts into a concise summary, providing you with a valuable study tool.

Vocabulary Exercises

The vocabulary exercises appear at the beginning of the exercise set for each section. These problems help you review previously learned vocabulary terms that you will use in solving the section exercises.

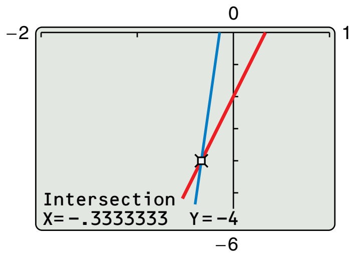
TECHNOLOGY You can use a graphing utility to check that a solution is reasonable. One way is to graph the left side of the equation, then graph the right side of the equation, and determine the point of intersection. For instance, in Example 2, if you graph the equations

$y_1 = 6(x - 1) + 4$ The left side

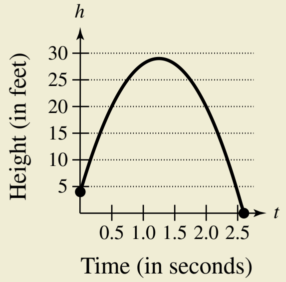
and

$y_2 = 3(7x + 1)$ The right side

in the same viewing window, they intersect at $x = -\frac{1}{3}$, as shown in the graph below.



92. HOW DO YOU SEE IT? The graph represents the height h of a projectile after t seconds.



(a) Explain why h is a function of t.

(b) Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

(c) Approximate the domain of h.

(d) Is t a function of h? Explain.

How Do You See It?

The How Do You See It? feature in each section presents a real-life exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

Project

The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at *LarsonPrecalculus.com.*

Chapter Summary

The Chapter Summary includes explanations and examples of the objectives taught in each chapter.

Instructor Resources

Annotated Instructor's Edition / ISBN-13: 978-1-337-28230-7
This is the complete student text plus point-of-use annotations for the instructor, including extra projects, classroom activities, teaching strategies, and additional examples. Answers to even-numbered text exercises, Vocabulary Checks, and Explorations are also provided.

Complete Solutions Manual (on instructor companion site)
This manual contains solutions to all exercises from the text, including Chapter Review Exercises and Chapter Tests, and Practice Tests with solutions.

Cengage Learning Testing Powered by Cognero (login.cengage.com)
CLT is a flexible online system that allows you to author, edit, and manage test bank content; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you want. This is available online via *www.cengage.com/login.*

Instructor Companion Site
Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online via *www.cengage.com/login.* Access and download PowerPoint® presentations, images, the instructor's manual, and more.

The Test Bank (on instructor companion site)
This contains text-specific multiple-choice and free response test forms.

Lesson Plans (on instructor companion site)
This manual provides suggestions for activities and lessons with notes on time allotment in order to ensure timeliness and efficiency during class.

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MindTap® is the digital learning solution that helps instructors engage and transform today's students into critical thinkers. Through paths of dynamic assignments and applications that you can personalize, real-time course analytics and an accessible reader, MindTap helps you turn cookie cutter into cutting edge, apathy into engagement, and memorizers into higher-level thinkers.

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Exclusively from Cengage Learning, Enhanced WebAssign combines the exceptional mathematics content that you know and love with the most powerful online homework solution, WebAssign. Enhanced WebAssign engages students with immediate feedback, rich tutorial content, and interactive, fully customizable e-books (YouBook), helping students to develop a deeper conceptual understanding of their subject matter. Quick Prep and Just In Time exercises provide opportunities for students to review prerequisite skills and content, both at the start of the course and at the beginning of each section. Flexible assignment options give instructors the ability to release assignments conditionally on the basis of students' prerequisite assignment scores. Visit us at **www.cengage.com/ewa** to learn more.

Student Resources

Student Study and Solutions Manual / ISBN-13: 978-1-337-29150-7
This guide offers step-by-step solutions for all odd-numbered text exercises, Chapter Tests, and Cumulative Tests. It also contains Practice Tests.

Note-Taking Guide / ISBN-13: 978-1-337-29151-4
This is an innovative study aid, in the form of a notebook organizer, that helps students develop a section-by-section summary of key concepts.

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To access additional course materials, please visit *www.cengagebrain.com.* At the *CengageBrain.com* home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where these resources can be found.

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MindTap® provides you with the tools you need to better manage your limited time—you can complete assignments whenever and wherever you are ready to learn with course material specially customized for you by your instructor and streamlined in one proven, easy-to-use interface. With an array of tools and apps—from note taking to flashcards—you'll get a true understanding of course concepts, helping you to achieve better grades and setting the groundwork for your future courses. This access code entitles you to one term of usage.

Enhanced WebAssign®
Enhanced WebAssign (assigned by the instructor) provides you with instant feedback on homework assignments. This online homework system is easy to use and includes helpful links to textbook sections, video examples, and problem-specific tutorials.

Acknowledgments

I would like to thank the many people who have helped me prepare the text and the supplements package. Their encouragement, criticisms, and suggestions have been invaluable.

Thank you to all of the instructors who took the time to review the changes in this edition and to provide suggestions for improving it. Without your help, this book would not be possible.

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On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil. If you have suggestions for improving this text, please feel free to write to me. Over the past two decades, I have received many useful comments from both instructors and students, and I value these comments very highly.

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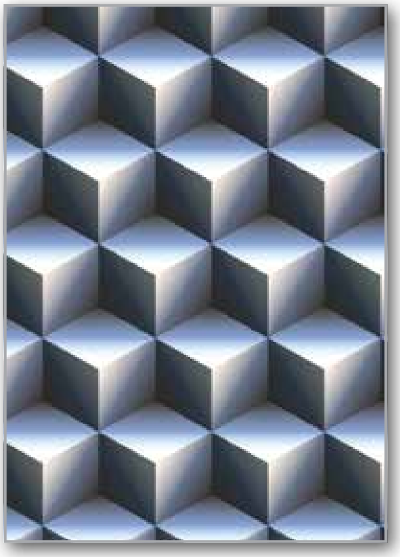
P Prerequisites



Computer Graphics *(page 56)*

Autocatalytic Chemical Reaction *(Exercise 84, page 40)*

Steel Beam Loading *(Exercise 81, page 33)*

Gallons of Water on Earth *(page 17)*

Change in Temperature *(page 7)*

Clockwise from top left, Suwit Ngaokaew/Shutterstock.com; Matt Antonino/Shutterstock.com;
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P.1 Review of Real Numbers and Their Properties

Real numbers can represent many real-life quantities. For example, in Exercises 49–52 on page 13, you will use real numbers to represent the federal surplus or deficit.

- Represent and classify real numbers.
- Order real numbers and use inequalities.
- Find the absolute values of real numbers and find the distance between two real numbers.
- Evaluate algebraic expressions.
- Use the basic rules and properties of algebra.

Real Numbers

Real numbers can describe quantities in everyday life such as age, miles per gallon, and population. Symbols such as

$$-5,\ 9,\ 0,\ \tfrac{4}{3},\ 0.666\ .\ .\ .\ ,\ 28.21,\ \sqrt{2},\ \pi,\ \text{and}\ \sqrt[3]{-32}$$

represent real numbers. Here are some important **subsets** (each member of a subset B is also a member of a set A) of the real numbers. The three dots, or *ellipsis points,* tell you that the pattern continues indefinitely.

$$\{1, 2, 3, 4, .\ .\ .\}$$ Set of natural numbers

$$\{0, 1, 2, 3, 4, .\ .\ .\}$$ Set of whole numbers

$$\{.\ .\ .\ , -3, -2, -1, 0, 1, 2, 3, .\ .\ .\}$$ Set of integers

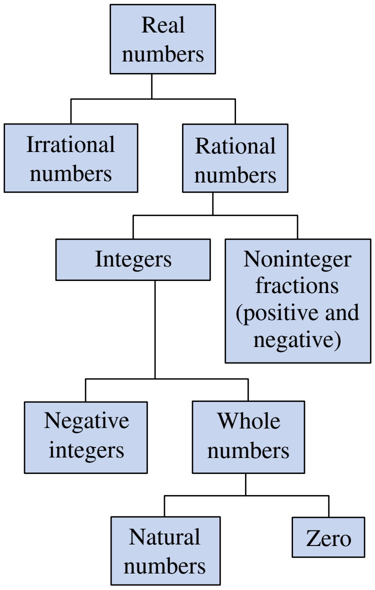
A real number is **rational** when it can be written as the ratio p/q of two integers, where $q \neq 0$. For example, the numbers

$$\tfrac{1}{3} = 0.3333\ .\ .\ . = 0.\overline{3},\ \tfrac{1}{8} = 0.125,\ \text{and}\ \tfrac{125}{111} = 1.126126\ .\ .\ . = 1.\overline{126}$$

are rational. The decimal representation of a rational number either repeats $\left(\text{as in } \tfrac{173}{55} = 3.1\overline{45}\right)$ or terminates $\left(\text{as in } \tfrac{1}{2} = 0.5\right)$. A real number that cannot be written as the ratio of two integers is **irrational.** The decimal representation of an irrational number neither terminates nor repeats. For example, the numbers

$$\sqrt{2} = 1.4142135\ldots \approx 1.41 \quad \text{and} \quad \pi = 3.1415926\ldots \approx 3.14$$

are irrational. (The symbol $\approx$ means "is approximately equal to.") Figure P.1 shows subsets of the real numbers and their relationships to each other.



Subsets of the real numbers
Figure P.1

EXAMPLE 1 Classifying Real Numbers

Determine which numbers in the set $\left\{-13, -\sqrt{5}, -1, -\tfrac{1}{3}, 0, \tfrac{5}{8}, \sqrt{2}, \pi, 7\right\}$ are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.

Solution

a. Natural numbers: $\{7\}$

b. Whole numbers: $\{0, 7\}$

c. Integers: $\{-13, -1, 0, 7\}$

d. Rational numbers: $\left\{-13, -1, -\tfrac{1}{3}, 0, \tfrac{5}{8}, 7\right\}$

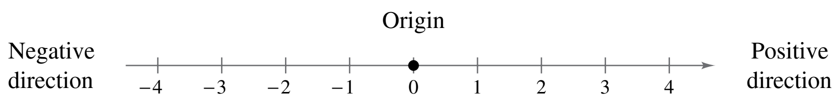
e. Irrational numbers: $\left\{-\sqrt{5}, \sqrt{2}, \pi\right\}$

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

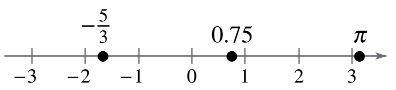
Repeat Example 1 for the set $\left\{-\pi, -\tfrac{1}{4}, \tfrac{6}{3}, \tfrac{1}{2}\sqrt{2}, -7.5, -1, 8, -22\right\}$.

Michael G Smith/Shutterstock.com

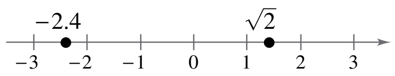
Real numbers are represented graphically on the **real number line.** When you draw a point on the real number line that corresponds to a real number, you are **plotting** the real number. The point representing 0 on the real number line is the **origin.** Numbers to the right of 0 are positive, and numbers to the left of 0 are negative, as shown in the figure below. The term **nonnegative** describes a number that is either positive or zero.



As the next two number lines illustrate, there is a *one-to-one correspondence* between real numbers and points on the real number line.



Every real number corresponds to exactly one point on the real number line.



Every point on the real number line corresponds to exactly one real number.

EXAMPLE 2 Plotting Points on the Real Number Line

Plot the real numbers on the real number line.

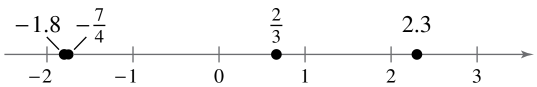
a. $-\dfrac{7}{4}$

b. 2.3

c. $\dfrac{2}{3}$

d. -1.8

Solution The figure below shows all four points.



a. The point representing the real number $-\frac{7}{4} = -1.75$ lies between -2 and -1, but closer to -2, on the real number line.

b. The point representing the real number 2.3 lies between 2 and 3, but closer to 2, on the real number line.

c. The point representing the real number $\frac{2}{3} = 0.666\ldots$ lies between 0 and 1, but closer to 1, on the real number line.

d. The point representing the real number -1.8 lies between -2 and -1, but closer to -2, on the real number line. Note that the point representing -1.8 lies slightly to the left of the point representing $-\frac{7}{4}$.

Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Plot the real numbers on the real number line.

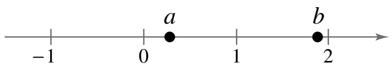
a. $\dfrac{5}{2}$ **b.** -1.6

c. $-\dfrac{3}{4}$ **d.** 0.7

Ordering Real Numbers

One important property of real numbers is that they are *ordered.*

> **Definition of Order on the Real Number Line**
>
> If a and b are real numbers, then a is *less than* b when $b - a$ is positive. The **inequality** $a < b$ denotes the **order** of a and b. This relationship can also be described by saying that b is *greater than* a and writing $b > a$. The inequality $a \le b$ means that a is *less than or equal to* b, and the inequality $b \ge a$ means that b is *greater than or equal to* a. The symbols $<$, $>$, $\le$, and $\ge$ are *inequality symbols.*



$a < b$ if and only if a lies to the left of b.
Figure P.2

Geometrically, this definition implies that $a < b$ if and only if a lies to the *left* of b on the real number line, as shown in Figure P.2.

EXAMPLE 3 Ordering Real Numbers

Place the appropriate inequality symbol ($<$ or $>$) between the pair of real numbers.

a. $-3, 0$ **b.** $-2, -4$ **c.** $\frac{1}{4}, \frac{1}{3}$

Solution

a. On the real number line, -3 lies to the left of 0, as shown in Figure P.3. So, you can say that -3 is *less than* 0, and write $-3 < 0$.

b. On the real number line, -2 lies to the right of -4, as shown in Figure P.4. So, you can say that -2 is *greater than* -4, and write $-2 > -4$.

c. On the real number line, $\frac{1}{4}$ lies to the left of $\frac{1}{3}$, as shown in Figure P.5. So, you can say that $\frac{1}{4}$ is *less than* $\frac{1}{3}$, and write $\frac{1}{4} < \frac{1}{3}$.

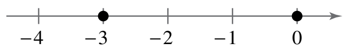


Figure P.3

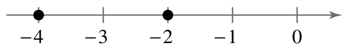


Figure P.4

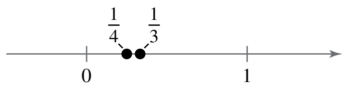


Figure P.5

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

Place the appropriate inequality symbol ($<$ or $>$) between the pair of real numbers.

a. $1, -5$ **b.** $\frac{3}{2}, 7$ **c.** $-\frac{2}{3}, -\frac{3}{4}$

EXAMPLE 4 Interpreting Inequalities

See LarsonPrecalculus.com for an interactive version of this type of example.

Describe the subset of real numbers that the inequality represents.

a. $x \le 2$ **b.** $-2 \le x < 3$

Solution

a. The inequality $x \le 2$ denotes all real numbers less than or equal to 2, as shown in Figure P.6.

b. The inequality $-2 \le x < 3$ means that $x \ge -2$ *and* $x < 3$. This "double inequality" denotes all real numbers between -2 and 3, including -2 but not including 3, as shown in Figure P.7.

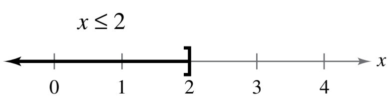


Figure P.6

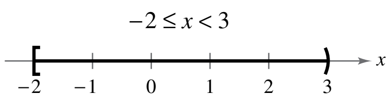


Figure P.7

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com*

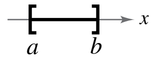
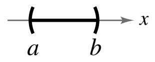
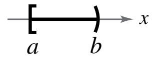
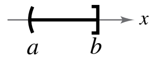
Describe the subset of real numbers that the inequality represents.

a. $x > -3$ **b.** $0 < x \le 4$

Inequalities can describe subsets of real numbers called **intervals.** In the bounded intervals below, the real numbers a and b are the **endpoints** of each interval. The endpoints of a closed interval are included in the interval, whereas the endpoints of an open interval are not included in the interval.

REMARK The reason that the four types of intervals at the right are called *bounded* is that each has a finite length. An interval that does not have a finite length is *unbounded* (see below).

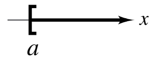
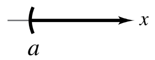
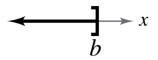
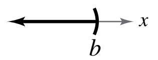
Bounded Intervals on the Real Number Line

Notation	Interval Type	Inequality	Graph
$[a, b]$	Closed	$a \leq x \leq b$	
(a, b)	Open	$a < x < b$	
$[a, b)$		$a \leq x < b$	
$(a, b]$		$a < x \leq b$	

The symbols ∞, **positive infinity,** and $-\infty$, **negative infinity,** do not represent real numbers. They are convenient symbols used to describe the unboundedness of an interval such as $(1, \infty)$ or $(-\infty, 3]$.

REMARK Whenever you write an interval containing ∞ or $-\infty$, always use a parenthesis and never a bracket next to these symbols. This is because ∞ and $-\infty$ are never included in the interval.

Unbounded Intervals on the Real Number Line

Notation	Interval Type	Inequality	Graph
$[a, \infty)$		$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$		$x \leq b$	
$(-\infty, b)$	Open	$x < b$	
$(-\infty, \infty)$	Entire real line	$-\infty < x < \infty$	

EXAMPLE 5 Interpreting Intervals

a. The interval $(-1, 0)$ consists of all real numbers greater than -1 and less than 0.

b. The interval $[2, \infty)$ consists of all real numbers greater than or equal to 2.

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Give a verbal description of the interval $[-2, 5)$.

EXAMPLE 6 Using Inequalities to Represent Intervals

a. The inequality $c \leq 2$ can represent the statement "c is at most 2."

b. The inequality $-3 < x \leq 5$ can represent "all x in the interval $(-3, 5]$."

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Use inequality notation to represent the statement "x is less than 4 and at least -2."

Absolute Value and Distance

The **absolute value** of a real number is its *magnitude,* or the distance between the origin and the point representing the real number on the real number line.

Definition of Absolute Value

If a is a real number, then the **absolute value** of a is

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}.$$

Notice in this definition that the absolute value of a real number is never negative. For example, if $a = -5$, then $|-5| = -(-5) = 5$. The absolute value of a real number is either positive or zero. Moreover, 0 is the only real number whose absolute value is 0. So, $|0| = 0$.

Properties of Absolute Values

1. $|a| \geq 0$

2. $|-a| = |a|$

3. $|ab| = |a||b|$

4. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \quad b \neq 0$

EXAMPLE 7 Finding Absolute Values

a. $|-15| = 15$ **b.** $\left|\dfrac{2}{3}\right| = \dfrac{2}{3}$

c. $|-4.3| = 4.3$ **d.** $-|-6| = -(6) = -6$

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Evaluate each expression.

a. $|1|$ **b.** $-\left|\dfrac{3}{4}\right|$ **c.** $\dfrac{2}{|-3|}$ **d.** $-|0.7|$

EXAMPLE 8 Evaluating an Absolute Value Expression

Evaluate $\dfrac{|x|}{x}$ for (a) $x > 0$ and (b) $x < 0$.

Solution

a. If $x > 0$, then x is positive and $|x| = x$. So, $\dfrac{|x|}{x} = \dfrac{x}{x} = 1$.

b. If $x < 0$, then x is negative and $|x| = -x$. So, $\dfrac{|x|}{x} = \dfrac{-x}{x} = -1$.

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Evaluate $\dfrac{|x+3|}{x+3}$ for (a) $x > -3$ and (b) $x < -3$.

The **Law of Trichotomy** states that for any two real numbers a and b, *precisely* one of three relationships is possible:

$a = b,\quad a < b,\quad \text{or}\quad a > b.$ Law of Trichotomy

EXAMPLE 9 Comparing Real Numbers

Place the appropriate symbol ($<$, $>$, or $=$) between the pair of real numbers.

a. $|-4| \;\square\; |3|$ **b.** $|-10| \;\square\; |10|$ **c.** $-|-7| \;\square\; |-7|$

Solution

a. $|-4| > |3|$ because $|-4| = 4$ and $|3| = 3$, and 4 is greater than 3.

b. $|-10| = |10|$ because $|-10| = 10$ and $|10| = 10$.

c. $-|-7| < |-7|$ because $-|-7| = -7$ and $|-7| = 7$, and -7 is less than 7.

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Place the appropriate symbol ($<$, $>$, or $=$) between the pair of real numbers.

a. $|-3| \;\square\; |4|$

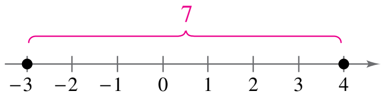
b. $-|-4| \;\square\; -|4|$

c. $|-3| \;\square\; -|-3|$

Absolute value can be used to find the distance between two points on the real number line. For example, the distance between -3 and 4 is

$$|-3 - 4| = |-7|$$
$$= 7$$

as shown in Figure P.8.



The distance between -3 and 4 is 7.
Figure P.8

Distance Between Two Points on the Real Number Line

Let a and b be real numbers. The **distance between a and b** is

$$d(a, b) = |b - a| = |a - b|.$$


One application of finding the distance between two points on the real number line is finding a change in temperature.

EXAMPLE 10 Finding a Distance

Find the distance between -25 and 13.

Solution

The distance between -25 and 13 is

$|-25 - 13| = |-38| = 38.$ Distance between -25 and 13

The distance can also be found as follows.

$|13 - (-25)| = |38| = 38$ Distance between -25 and 13

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a. Find the distance between 35 and -23.

b. Find the distance between -35 and -23.

c. Find the distance between 35 and 23.

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Algebraic Expressions

One characteristic of algebra is the use of letters to represent numbers. The letters are **variables,** and combinations of letters and numbers are **algebraic expressions.** Here are a few examples of algebraic expressions.

$$5x, \quad 2x - 3, \quad \frac{4}{x^2 + 2}, \quad 7x + y$$

Definition of an Algebraic Expression

An **algebraic expression** is a collection of letters **(variables)** and real numbers **(constants)** combined using the operations of addition, subtraction, multiplication, division, and exponentiation.

The **terms** of an algebraic expression are those parts that are separated by *addition.* For example, $x^2 - 5x + 8 = x^2 + (-5x) + 8$ has three terms: x^2 and $-5x$ are the **variable terms** and 8 is the **constant term.** For terms such as x^2, $-5x$, and 8, the numerical factor is the **coefficient.** Here, the coefficients are 1, -5, and 8.

EXAMPLE 11 Identifying Terms and Coefficients

	Algebraic Expression	Terms	Coefficients
a.	$5x - \frac{1}{7}$	$5x, -\frac{1}{7}$	$5, -\frac{1}{7}$
b.	$2x^2 - 6x + 9$	$2x^2, -6x, 9$	$2, -6, 9$
c.	$\frac{3}{x} + \frac{1}{2}x^4 - y$	$\frac{3}{x}, \frac{1}{2}x^4, -y$	$3, \frac{1}{2}, -1$

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Identify the terms and coefficients of $-2x + 4$.

The **Substitution Principle** states, "If $a = b$, then b can replace a in any expression involving a." Use the Substitution Principle to **evaluate** an algebraic expression by substituting numerical values for each of the variables in the expression. The next example illustrates this.

EXAMPLE 12 Evaluating Algebraic Expressions

	Expression	Value of Variable	Substitute.	Value of Expression
a.	$-3x + 5$	$x = 3$	$-3(3) + 5$	$-9 + 5 = -4$
b.	$3x^2 + 2x - 1$	$x = -1$	$3(-1)^2 + 2(-1) - 1$	$3 - 2 - 1 = 0$
c.	$\frac{2x}{x + 1}$	$x = -3$	$\frac{2(-3)}{-3 + 1}$	$\frac{-6}{-2} = 3$

Note that you must substitute the value for *each* occurrence of the variable.

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Evaluate $4x - 5$ when $x = 0$.

Basic Rules of Algebra

There are four arithmetic operations with real numbers: *addition, multiplication, subtraction,* and *division,* denoted by the symbols $+$, $\times$ or $\cdot$, $-$, and $\div$ or $/$, respectively. Of these, addition and multiplication are the two primary operations. Subtraction and division are the inverse operations of addition and multiplication, respectively.

Definitions of Subtraction and Division

Subtraction: Add the opposite.

$$a - b = a + (-b)$$

Division: Multiply by the reciprocal.

If $b \neq 0$, then $a/b = a\left(\frac{1}{b}\right) = \frac{a}{b}$.

In these definitions, $-b$ is the **additive inverse** (or opposite) of b, and $1/b$ is the **multiplicative inverse** (or reciprocal) of b. In the fractional form a/b, a is the **numerator** of the fraction and b is the **denominator.**

The properties of real numbers below are true for variables and algebraic expressions as well as for real numbers, so they are often called the **Basic Rules of Algebra.** Formulate a verbal description of each of these properties. For example, the first property states that *the order in which two real numbers are added does not affect their sum.*

Basic Rules of Algebra

Let a, b, and c be real numbers, variables, or algebraic expressions.

Property		**Example**
Commutative Property of Addition:	$a + b = b + a$	$4x + x^2 = x^2 + 4x$
Commutative Property of Multiplication:	$ab = ba$	$(4 - x)x^2 = x^2(4 - x)$
Associative Property of Addition:	$(a + b) + c = a + (b + c)$	$(x + 5) + x^2 = x + (5 + x^2)$
Associative Property of Multiplication:	$(ab)c = a(bc)$	$(2x \cdot 3y)(8) = (2x)(3y \cdot 8)$
Distributive Properties:	$a(b + c) = ab + ac$	$3x(5 + 2x) = 3x \cdot 5 + 3x \cdot 2x$
	$(a + b)c = ac + bc$	$(y + 8)y = y \cdot y + 8 \cdot y$
Additive Identity Property:	$a + 0 = a$	$5y^2 + 0 = 5y^2$
Multiplicative Identity Property:	$a \cdot 1 = a$	$(4x^2)(1) = 4x^2$
Additive Inverse Property:	$a + (-a) = 0$	$5x^3 + (-5x^3) = 0$
Multiplicative Inverse Property:	$a \cdot \frac{1}{a} = 1, \quad a \neq 0$	$(x^2 + 4)\left(\frac{1}{x^2 + 4}\right) = 1$

Subtraction is defined as "adding the opposite," so the Distributive Properties are also true for subtraction. For example, the "subtraction form" of $a(b + c) = ab + ac$ is $a(b - c) = ab - ac$. Note that the operations of subtraction and division are neither commutative nor associative. The examples

$$7 - 3 \neq 3 - 7 \quad \text{and} \quad 20 \div 4 \neq 4 \div 20$$

show that subtraction and division are not commutative. Similarly

$$5 - (3 - 2) \neq (5 - 3) - 2 \quad \text{and} \quad 16 \div (4 \div 2) \neq (16 \div 4) \div 2$$

demonstrate that subtraction and division are not associative.

EXAMPLE 13 Identifying Rules of Algebra

Identify the rule of algebra illustrated by the statement.

a. $(5x^3)2 = 2(5x^3)$ **b.** $(4x + 3) - (4x + 3) = 0$

c. $7x \cdot \frac{1}{7x} = 1, \quad x \neq 0$ **d.** $(2 + 5x^2) + x^2 = 2 + (5x^2 + x^2)$

Solution

a. This statement illustrates the Commutative Property of Multiplication. In other words, you obtain the same result whether you multiply $5x^3$ by 2, or 2 by $5x^3$.

b. This statement illustrates the Additive Inverse Property. In terms of subtraction, this property states that when any expression is subtracted from itself, the result is 0.

c. This statement illustrates the Multiplicative Inverse Property. Note that x must be a nonzero number. The reciprocal of x is undefined when x is 0.

d. This statement illustrates the Associative Property of Addition. In other words, to form the sum $2 + 5x^2 + x^2$, it does not matter whether 2 and $5x^2$, or $5x^2$ and x^2 are added first.

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Identify the rule of algebra illustrated by the statement.

a. $x + 9 = 9 + x$ **b.** $5(x^3 \cdot 2) = (5x^3)2$ **c.** $(2 + 5x^2)y^2 = 2 \cdot y^2 + 5x^2 \cdot y^2$

REMARK Notice the difference between the *opposite of a number* and a *negative number.* If a is already negative, then its opposite, $-a$, is positive. For example, if $a = -5$, then

$$-a = -(-5) = 5.$$

Properties of Negation and Equality

Let a, b, and c be real numbers, variables, or algebraic expressions.

Property	Example
1. $(-1)a = -a$	$(-1)7 = -7$
2. $-(-a) = a$	$-(-6) = 6$
3. $(-a)b = -(ab) = a(-b)$	$(-5)3 = -(5 \cdot 3) = 5(-3)$
4. $(-a)(-b) = ab$	$(-2)(-x) = 2x$
5. $-(a + b) = (-a) + (-b)$	$-(x + 8) = (-x) + (-8) = -x - 8$
6. If $a = b$, then $a \pm c = b \pm c$.	$\frac{1}{2} + 3 = 0.5 + 3$
7. If $a = b$, then $ac = bc$.	$4^2 \cdot 2 = 16 \cdot 2$
8. If $a \pm c = b \pm c$, then $a = b$.	$1.4 - 1 = \frac{7}{5} - 1 \implies 1.4 = \frac{7}{5}$
9. If $ac = bc$ and $c \neq 0$, then $a = b$.	$3x = 3 \cdot 4 \implies x = 4$

REMARK The "or" in the Zero-Factor Property includes the possibility that either or both factors may be zero. This is an *inclusive or,* and it is generally the way the word "or" is used in mathematics.

Properties of Zero

Let a and b be real numbers, variables, or algebraic expressions.

1. $a + 0 = a$ and $a - 0 = a$ **2.** $a \cdot 0 = 0$

3. $\frac{0}{a} = 0, \quad a \neq 0$ **4.** $\frac{a}{0}$ is undefined.

5. Zero-Factor Property: If $ab = 0$, then $a = 0$ or $b = 0$.

REMARK In Property 1, the phrase "if and only if" implies two statements. One statement is: If $a/b = c/d$, then $ad = bc$. The other statement is: If $ad = bc$, where $b \neq 0$ and $d \neq 0$, then $a/b = c/d$.

Properties and Operations of Fractions

Let a, b, c, and d be real numbers, variables, or algebraic expressions such that $b \neq 0$ and $d \neq 0$.

1. **Equivalent Fractions:** $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.
2. **Rules of Signs:** $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ and $\frac{-a}{-b} = \frac{a}{b}$
3. **Generate Equivalent Fractions:** $\frac{a}{b} = \frac{ac}{bc}$, $\quad c \neq 0$
4. **Add or Subtract with Like Denominators:** $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$
5. **Add or Subtract with Unlike Denominators:** $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$
6. **Multiply Fractions:** $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
7. **Divide Fractions:** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, $\quad c \neq 0$

EXAMPLE 14 Properties and Operations of Fractions

a. $\frac{x}{5} = \frac{3 \cdot x}{3 \cdot 5} = \frac{3x}{15}$ **b.** $\frac{7}{x} \div \frac{3}{2} = \frac{7}{x} \cdot \frac{2}{3} = \frac{14}{3x}$

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a. Multiply fractions: $\frac{3}{5} \cdot \frac{x}{6}$ **b.** Add fractions: $\frac{x}{10} + \frac{2x}{5}$

REMARK The number 1 is neither prime nor composite.

If a, b, and c are integers such that $ab = c$, then a and b are **factors** or **divisors** of c. A **prime number** is an integer that has exactly two positive factors—itself and 1—such as 2, 3, 5, 7, and 11. The numbers 4, 6, 8, 9, and 10 are **composite** because each can be written as the product of two or more prime numbers. The **Fundamental Theorem of Arithmetic** states that every positive integer greater than 1 is a prime number or can be written as the product of prime numbers in precisely one way (disregarding order). For example, the *prime factorization* of 24 is $24 = 2 \cdot 2 \cdot 2 \cdot 3$.

Summarize (Section P.1)

1. Explain how to represent and classify real numbers *(pages 2 and 3)*. For examples of representing and classifying real numbers, see Examples 1 and 2.
2. Explain how to order real numbers and use inequalities *(pages 4 and 5)*. For examples of ordering real numbers and using inequalities, see Examples 3–6.
3. State the definition of the absolute value of a real number *(page 6)*. For examples of using absolute value, see Examples 7–10.
4. Explain how to evaluate an algebraic expression *(page 8)*. For examples involving algebraic expressions, see Examples 11 and 12.
5. State the basic rules and properties of algebra *(pages 9–11)*. For examples involving the basic rules and properties of algebra, see Examples 13 and 14.

P.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. The decimal representation of an ________ number neither terminates nor repeats.
2. The point representing 0 on the real number line is the ________.
3. The distance between the origin and a point representing a real number on the real number line is the ________ ________ of the real number.
4. A number that can be written as the product of two or more prime numbers is a ________ number.
5. The ________ of an algebraic expression are those parts that are separated by addition.
6. The ________ ________ states that if $ab = 0$, then $a = 0$ or $b = 0$.

Skills and Applications

Classifying Real Numbers **In Exercises 7–10, determine which numbers in the set are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.**

7. $\left\{-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{2}, 0, 1, -4, 2, -11\right\}$

8. $\left\{\sqrt{5}, -7, -\frac{7}{3}, 0, 3.14, \frac{5}{4}, -3, 12, 5\right\}$

9. $\{2.01, 0.\overline{6}, -13, 0.010110111 \ldots, 1, -6\}$

10. $\left\{25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 7, -11.1, 13\right\}$

Plotting Points on the Real Number Line **In Exercises 11 and 12, plot the real numbers on the real number line.**

11. (a) 3 (b) $\frac{7}{2}$ (c) $-\frac{5}{2}$ (d) -5.2

12. (a) 8.5 (b) $\frac{4}{3}$ (c) -4.75 (d) $-\frac{8}{3}$

Plotting and Ordering Real Numbers **In Exercises 13–16, plot the two real numbers on the real number line. Then place the appropriate inequality symbol ($<$ or $>$) between them.**

13. $-4, -8$

14. $1, \frac{16}{3}$

15. $\frac{5}{6}, \frac{2}{3}$

16. $-\frac{8}{7}, -\frac{3}{7}$

Interpreting an Inequality or an Interval **In Exercises 17–24, (a) give a verbal description of the subset of real numbers represented by the inequality or the interval, (b) sketch the subset on the real number line, and (c) state whether the subset is bounded or unbounded.**

17. $x \le 5$

18. $x < 0$

19. $-2 < x < 2$

20. $0 < x \le 6$

21. $[4, \infty)$

22. $(-\infty, 2)$

23. $[-5, 2)$

24. $(-1, 2]$

Using Inequality and Interval Notation **In Exercises 25–28, use inequality notation and interval notation to describe the set.**

25. y is nonnegative.

26. y is no more than 25.

27. t is at least 10 and at most 22.

28. k is less than 5 but no less than -3.

Evaluating an Absolute Value Expression **In Exercises 29–38, evaluate the expression.**

29. $|-10|$

30. $|0|$

31. $|3 - 8|$

32. $|6 - 2|$

33. $|-1| - |-2|$

34. $-3 - |-3|$

35. $5|-5|$

36. $-4|-4|$

37. $\dfrac{|x+2|}{x+2}, \quad x < -2$

38. $\dfrac{|x-1|}{x-1}, \quad x > 1$

Comparing Real Numbers **In Exercises 39–42, place the appropriate symbol ($<$, $>$, or $=$) between the pair of real numbers.**

39. $|-4| \ \square \ |4|$

40. $-5 \ \square \ -|5|$

41. $-|-6| \ \square \ |-6|$

42. $-|-2| \ \square \ -|2|$

Finding a Distance **In Exercises 43–46, find the distance between a and b.**

43. $a = 126, b = 75$

44. $a = -20, b = 30$

45. $a = -\frac{5}{2}, b = 0$

46. $a = -\frac{1}{4}, b = -\frac{11}{4}$

Using Absolute Value Notation **In Exercises 47 and 48, use absolute value notation to represent the situation.**

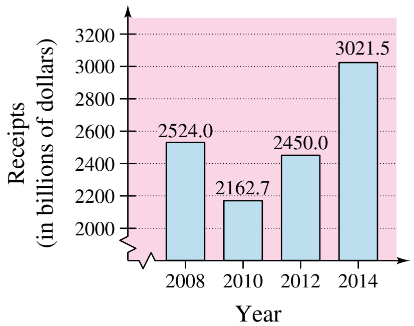
47. The distance between x and 5 is no more than 3.

48. The distance between x and -10 is at least 6.

The symbol ▩ and a red exercise number indicates that a video solution can be seen at *CalcView.com*.

Federal Deficit

In Exercises 49–52, use the bar graph, which shows the receipts of the federal government (in billions of dollars) for selected years from 2008 through 2014. In each exercise, you are given the expenditures of the federal government. Find the magnitude of the surplus or deficit for the year. *(Source: U.S. Office of Management and Budget)*



	Year	Receipts, R	Expenditures, E	$\lvert R - E \rvert$
49.	2008		\$2982.5 billion	
50.	2010		\$3457.1 billion	
51.	2012		\$3537.0 billion	
52.	2014		\$3506.1 billion	

Identifying Terms and Coefficients

In Exercises 53–58, identify the terms. Then identify the coefficients of the variable terms of the expression.

53. $7x + 4$ **54.** $2x - 3$

55. $6x^3 - 5x$ **56.** $4x^3 + 0.5x - 5$

57. $3\sqrt{3}x^2 + 1$ **58.** $2\sqrt{2}x^2 - 3$

Evaluating an Algebraic Expression

In Exercises 59–64, evaluate the expression for each value of x. (If not possible, state the reason.)

59. $4x - 6$ (a) $x = -1$ (b) $x = 0$

60. $9 - 7x$ (a) $x = -3$ (b) $x = 3$

61. $x^2 - 3x + 2$ (a) $x = 0$ (b) $x = -1$

62. $-x^2 + 5x - 4$ (a) $x = -1$ (b) $x = 1$

63. $\dfrac{x+1}{x-1}$ (a) $x = 1$ (b) $x = -1$

64. $\dfrac{x-2}{x+2}$ (a) $x = 2$ (b) $x = -2$

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Identifying Rules of Algebra

In Exercises 65–68, identify the rule(s) of algebra illustrated by the statement.

65. $\dfrac{1}{h+6}(h + 6) = 1, \quad h \neq -6$

66. $(x + 3) - (x + 3) = 0$

67. $x(3y) = (x \cdot 3)y = (3x)y$

68. $\frac{1}{7}(7 \cdot 12) = \left(\frac{1}{7} \cdot 7\right)12 = 1 \cdot 12 = 12$

Operations with Fractions

In Exercises 69–72, perform the operation. (Write fractional answers in simplest form.)

69. $\dfrac{2x}{3} - \dfrac{x}{4}$ **70.** $\dfrac{3x}{4} + \dfrac{x}{5}$

71. $\dfrac{3x}{10} \cdot \dfrac{5}{6}$ **72.** $\dfrac{2x}{3} \div \dfrac{6}{7}$

Exploration

True or False?

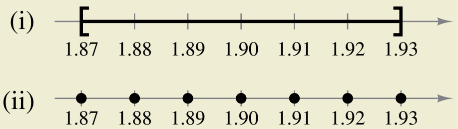
In Exercises 73–75, determine whether the statement is true or false. Justify your answer.

73. Every nonnegative number is positive.

74. If $a > 0$ and $b < 0$, then $ab > 0$.

75. If $a < 0$ and $b < 0$, then $ab > 0$.

76. **HOW DO YOU SEE IT?** Match each description with its graph. Which types of real numbers shown in Figure P.1 on page 2 may be included in a range of prices? a range of lengths? Explain.



(a) The price of an item is within \$0.03 of \$1.90.

(b) The distance between the prongs of an electric plug may not differ from 1.9 centimeters by more than 0.03 centimeter.

77. Conjecture

(a) Use a calculator to complete the table.

n	0.0001	0.01	1	100	10,000
$\dfrac{5}{n}$					

(b) Use the result from part (a) to make a conjecture about the value of $5/n$ as n (i) approaches 0, and (ii) increases without bound.

P.2 Exponents and Radicals

Real numbers and algebraic expressions are often written with exponents and radicals. For example, in Exercise 69 on page 25, you will use an expression involving rational exponents to find the times required for a funnel to empty for different water heights.

- **Use properties of exponents.**
- **Use scientific notation to represent real numbers.**
- **Use properties of radicals.**
- **Simplify and combine radical expressions.**
- **Rationalize denominators and numerators.**
- **Use properties of rational exponents.**

Integer Exponents and Their Properties

Repeated *multiplication* can be written in **exponential form.**

Repeated Multiplication	Exponential Form
$a \cdot a \cdot a \cdot a \cdot a$	a^5
$(-4)(-4)(-4)$	$(-4)^3$
$(2x)(2x)(2x)(2x)$	$(2x)^4$

Exponential Notation

If a is a real number and n is a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

where n is the **exponent** and a is the **base.** You read a^n as "a to the nth **power.**"

An exponent can also be negative or zero. Properties 3 and 4 below show how to use negative and zero exponents.

Properties of Exponents

Let a and b be real numbers, variables, or algebraic expressions, and let m and n be integers. (All denominators and bases are nonzero.)

Property	Example
1. $a^m a^n = a^{m+n}$	$3^2 \cdot 3^4 = 3^{2+4} = 3^6 = 729$
2. $\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{x^7}{x^4} = x^{7-4} = x^3$
3. $a^{-n} = \dfrac{1}{a^n} = \left(\dfrac{1}{a}\right)^n$	$y^{-4} = \dfrac{1}{y^4} = \left(\dfrac{1}{y}\right)^4$
4. $a^0 = 1$	$(x^2 + 1)^0 = 1$
5. $(ab)^m = a^m b^m$	$(5x)^3 = 5^3x^3 = 125x^3$
6. $(a^m)^n = a^{mn}$	$(y^3)^{-4} = y^{3(-4)} = y^{-12} = \dfrac{1}{y^{12}}$
7. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$\left(\dfrac{2}{x}\right)^3 = \dfrac{2^3}{x^3} = \dfrac{8}{x^3}$
8. $\lvert a^2 \rvert = \lvert a \rvert^2 = a^2$	$\lvert(-2)^2\rvert = \lvert -2 \rvert^2 = 2^2 = 4 = (-2)^2$

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The properties of exponents listed on the preceding page apply to *all* integers m and n, not just to positive integers, as shown in Examples 1–4.

It is important to recognize the difference between expressions such as $(-2)^4$ and -2^4. In $(-2)^4$, the parentheses tell you that the exponent applies to the negative sign as well as to the 2, but in $-2^4 = -(2^4)$, the exponent applies only to the 2. So, $(-2)^4 = 16$ and $-2^4 = -16$.

EXAMPLE 1 Evaluating Exponential Expressions

a. $(-5)^2 = (-5)(-5) = 25$ — Negative sign is part of the base.

b. $-5^2 = -(5)(5) = -25$ — Negative sign is *not* part of the base.

c. $2 \cdot 2^4 = 2^{1+4} = 2^5 = 32$ — Property 1

d. $\dfrac{4^4}{4^6} = 4^{4-6} = 4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$ — Properties 2 and 3

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Evaluate each expression.

a. -3^4 **b.** $(-3)^4$

c. $3^2 \cdot 3$ **d.** $\dfrac{3^5}{3^8}$

TECHNOLOGY When using a calculator to evaluate exponential expressions, it is important to know when to use parentheses because the calculator follows the order of operations. For example, here is how you would evaluate $(-2)^4$ on a graphing utility.

((−) 2) ^ 4 ENTER

The display will be 16. If you omit the parentheses, the display will be -16.

EXAMPLE 2 Evaluating Algebraic Expressions

Evaluate each algebraic expression when $x = 3$.

a. $5x^{-2}$ **b.** $\dfrac{1}{3}(-x)^3$

Solution

a. When $x = 3$, the expression $5x^{-2}$ has a value of

$$5x^{-2} = 5(3)^{-2} = \frac{5}{3^2} = \frac{5}{9}.$$

b. When $x = 3$, the expression $\frac{1}{3}(-x)^3$ has a value of

$$\frac{1}{3}(-x)^3 = \frac{1}{3}(-3)^3 = \frac{1}{3}(-27) = -9.$$

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Evaluate each algebraic expression when $x = 4$.

a. $-x^{-2}$ **b.** $\dfrac{1}{4}(-x)^4$

EXAMPLE 3 Using Properties of Exponents

Use the properties of exponents to simplify each expression.

a. $(-3ab^4)(4ab^{-3})$ **b.** $(2xy^2)^3$ **c.** $3a(-4a^2)^0$ **d.** $\left(\frac{5x^3}{y}\right)^2$

Solution

a. $(-3ab^4)(4ab^{-3}) = (-3)(4)(a)(a)(b^4)(b^{-3}) = -12a^2b$

b. $(2xy^2)^3 = 2^3(x)^3(y^2)^3 = 8x^3y^6$

c. $3a(-4a^2)^0 = 3a(1) = 3a$

d. $\left(\frac{5x^3}{y}\right)^2 = \frac{5^2(x^3)^2}{y^2} = \frac{25x^6}{y^2}$

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Use the properties of exponents to simplify each expression.

a. $(2x^{-2}y^3)(-x^4y)$ **b.** $(4a^2b^3)^0$ **c.** $(-5z)^3(z^2)$ **d.** $\left(\frac{3x^4}{x^2y^2}\right)^2$

REMARK Rarely in algebra is there only one way to solve a problem. Do not be concerned when the steps you use to solve a problem are not exactly the same as the steps presented in this text. It is important to use steps that you understand and, of course, steps that are justified by the rules of algebra. For example, the fractional form of Property 3 is

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m.$$

So, you might prefer the steps below for Example 4(d).

$$\left(\frac{3x^2}{y}\right)^{-2} = \left(\frac{y}{3x^2}\right)^2 = \frac{y^2}{9x^4}$$

EXAMPLE 4 Rewriting with Positive Exponents

a. $x^{-1} = \frac{1}{x}$ Property 3

b. $\frac{1}{3x^{-2}} = \frac{1(x^2)}{3}$ Property 3 (The exponent -2 does not apply to 3.)

$= \frac{x^2}{3}$ Simplify.

c. $\frac{12a^3b^{-4}}{4a^{-2}b} = \frac{12a^3 \cdot a^2}{4b \cdot b^4}$ Property 3

$= \frac{3a^5}{b^5}$ Property 1

d. $\left(\frac{3x^2}{y}\right)^{-2} = \frac{3^{-2}(x^2)^{-2}}{y^{-2}}$ Properties 5 and 7

$= \frac{3^{-2}x^{-4}}{y^{-2}}$ Property 6

$= \frac{y^2}{3^2x^4}$ Property 3

$= \frac{y^2}{9x^4}$ Simplify.

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Rewrite each expression with positive exponents. Simplify, if possible.

a. $2a^{-2}$ **b.** $\frac{3a^{-3}b^4}{15ab^{-1}}$

c. $\left(\frac{x}{10}\right)^{-1}$ **d.** $(-2x^2)^3(4x^3)^{-1}$